

## A GEOMETRIC APPROACH TO CONFIDENCE SETS FOR RATIOS: FIELLER'S THEOREM, GENERALIZATIONS, AND BOOTSTRAP

Ulrike von Luxburg and Volker H. Franz

*Max Planck Institute for Biological Cybernetics and Justus-Liebig-Universität Giessen*

### Supplementary Material

In this supplementary document we present detailed evaluations of all simulations conducted in Section 5.3. of the paper “A Geometric Approach to Confidence Sets for Ratios: Fieller’s Theorem, Generalizations, and Bootstrap” (called the main paper below). For the general setup of the simulations, please see Section 5.3. of the main paper.

In general, the simulations compare the performance of three methods of computing confidence sets for ratios. As we conducted a huge number of different simulations, numeric tables would become rather impractical and tedious to parse. Hence, we decided to present all evaluations in a more compact graphical form. For each simulation we show two color figures: one figure which compares the empirical coverage of the different methods, and one figure which compares the number of bounded confidence sets for the different methods.

One “row” of figures always refers to the same parameter settings. Those settings are given as the title for each row, for example:  $X \sim \text{normal}$ ,  $Y \sim \text{exponential}$ ,  $n = 100$ , nominal level 0.90. For details on how we define the distributions please see the main paper. The parameter  $n$  represents the sample size, the “nominal level 0.90” refers to the nominal confidence level  $1 - \alpha$ . Each experiment has been repeated  $R = 1000$  times, and the reported values are means over those repetitions.

Each of the three subfigures in one row corresponds to one particular method of computing the confidence sets: our geometric bootstrap method, Hwang’s bootstrap method, and Fieller’s classical method. For definitions and implementation of the different methods please see the main paper. Each subfigure shows the results in a color-coded “table”. The two axes of the table correspond to the parameters of the distributions of  $X$  and  $Y$ , respectively (indicated by the axes labels in the figures; for details on distributional parameters see main paper). For each combination of parameters, we report the corresponding value in the

experiment as a color-coded square. The scale of the color-code is indicated on the right hand side of each row. Note that color scales change between rows, but are constant within rows.

We report two types of results:

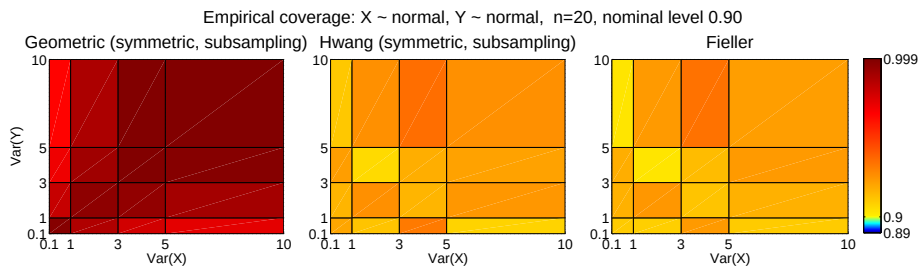
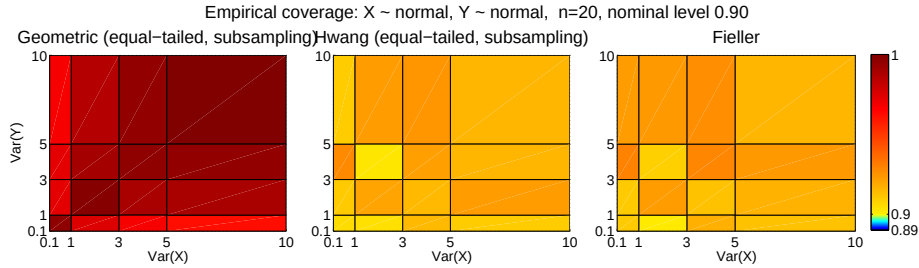
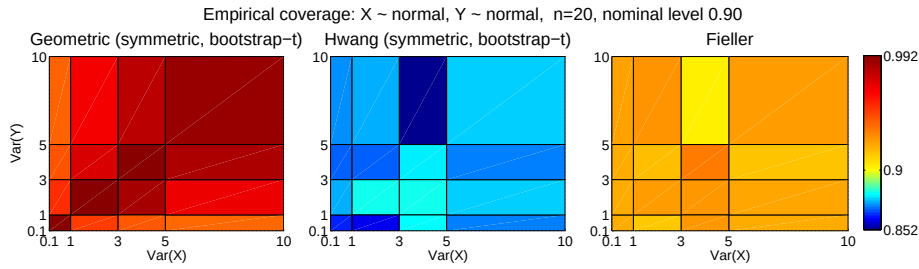
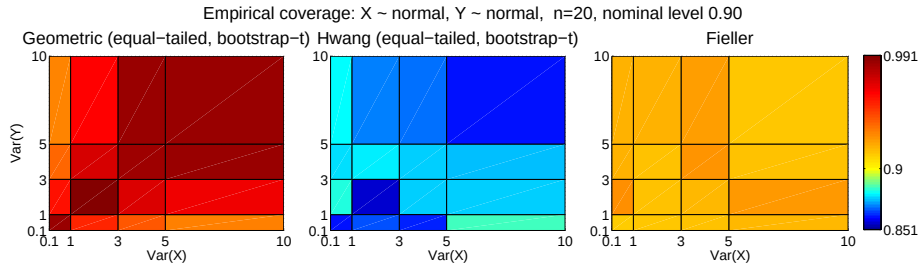
*The empirical coverage.* This shows how well the confidence sets achieve their nominal coverage level. In those figures, the colors are not on a “linear scale”. Instead, the nominal confidence level 0.90 is always depicted in yellow, red colors depict conservative and green/blue colors liberal confidence sets.

*The fraction of bounded confidence sets.* Here we report the fraction of repetitions for which the confidence sets were bounded of the form  $[l, u]$ . Here, color scales are “linear”.

For a summary of the conclusions which can be drawn from all simulations please see the main paper.

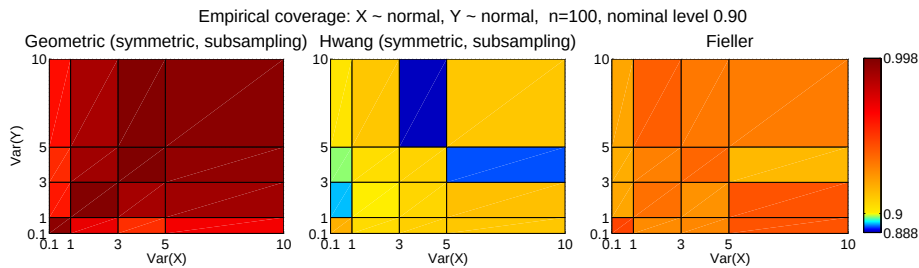
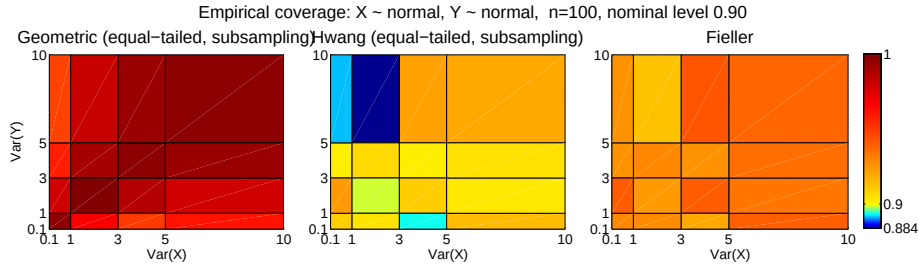
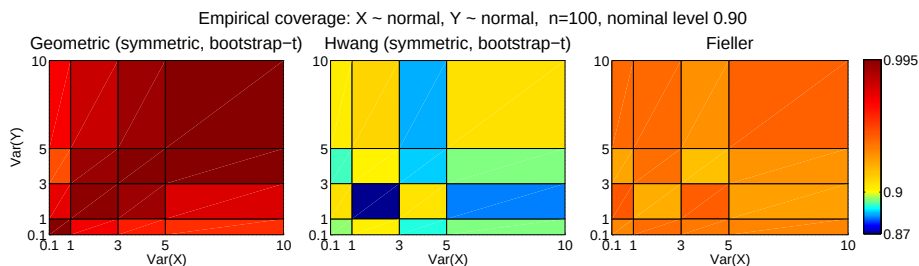
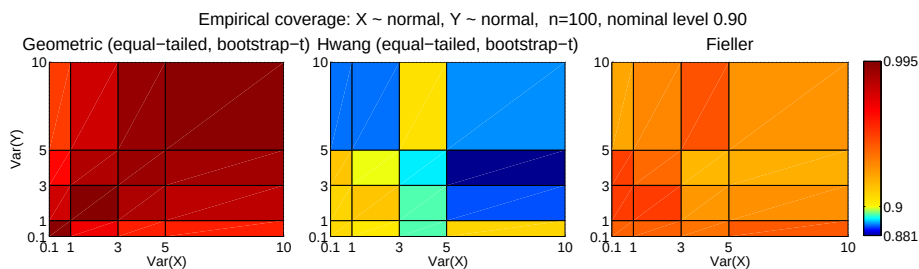
## Empirical coverage

$X \sim \text{normal}, Y \sim \text{normal}, n=20$



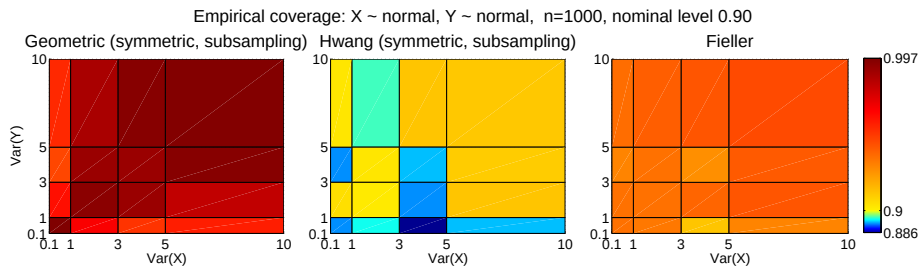
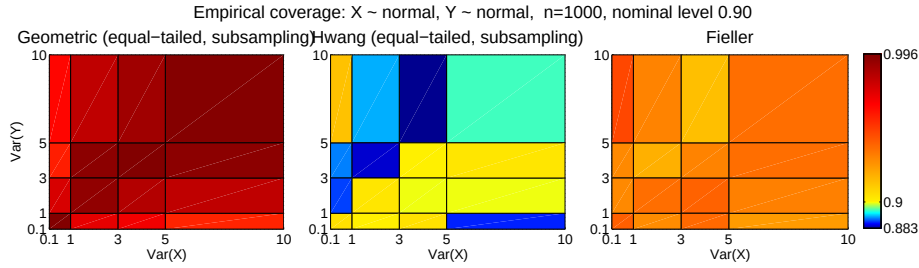
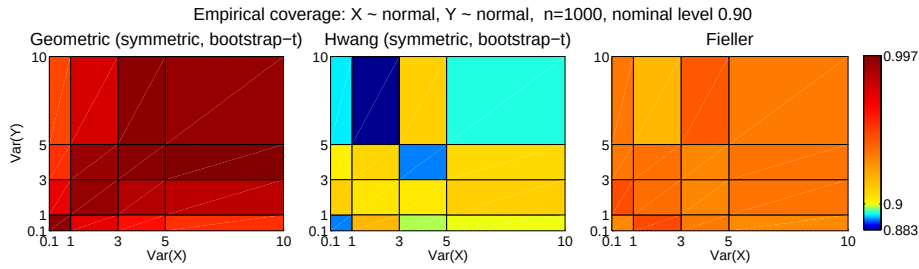
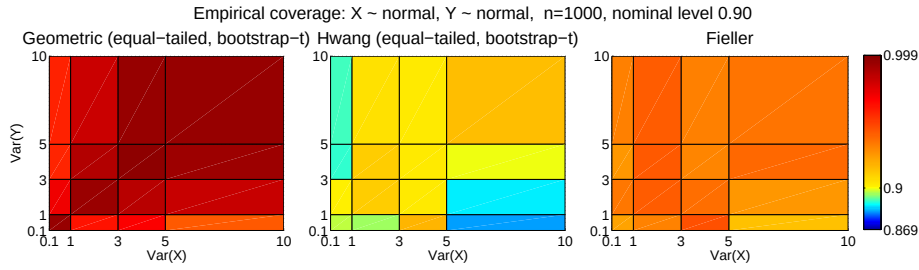
## Empirical coverage

### $X \sim \text{normal}$ , $Y \sim \text{normal}$ , $n=100$



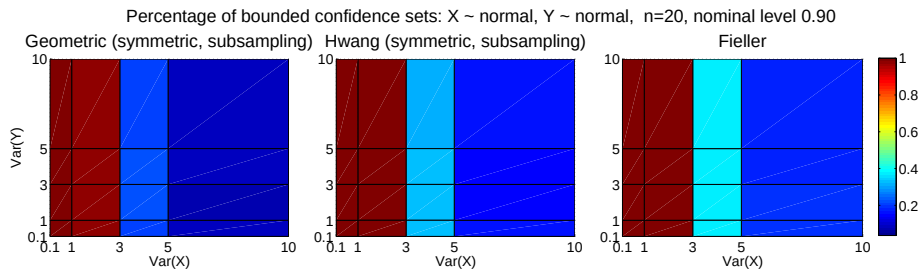
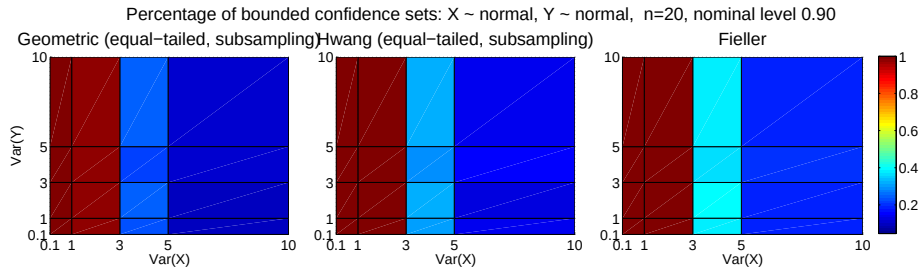
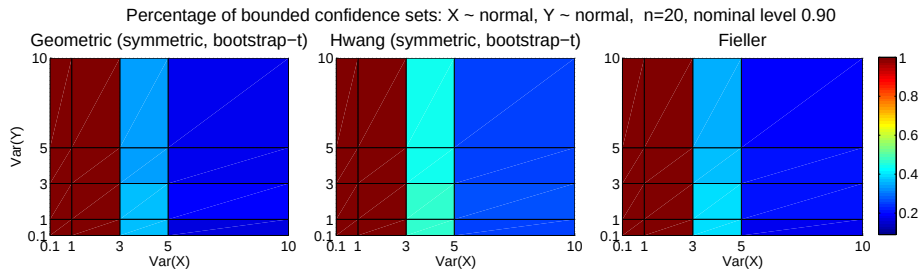
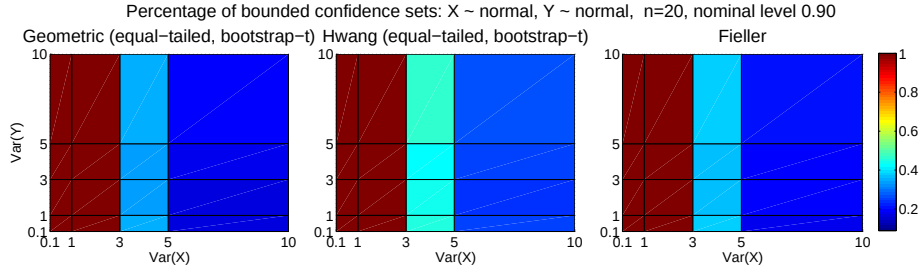
## Empirical coverage

$X \sim \text{normal}, Y \sim \text{normal}, n=1000$



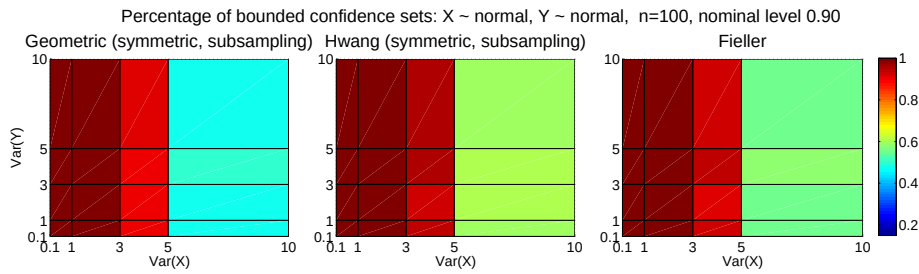
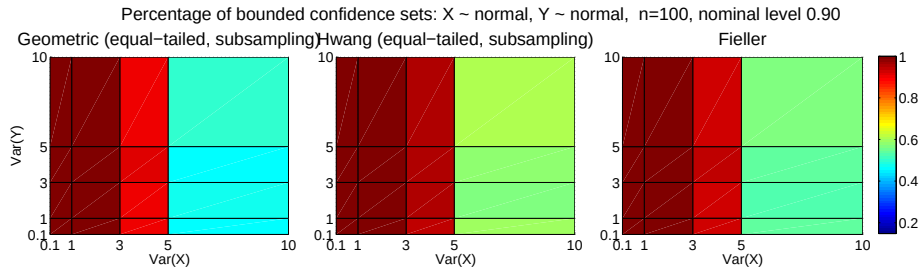
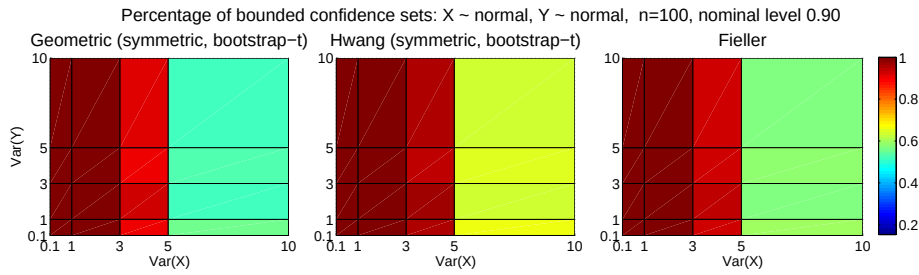
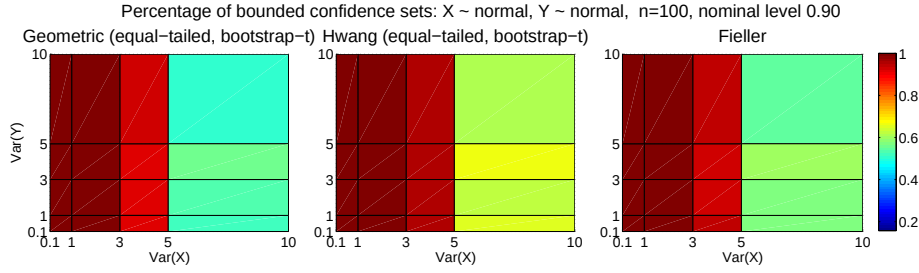
## Number of bounded confidence sets

$X \sim \text{normal}$ ,  $Y \sim \text{normal}$ ,  $n=20$



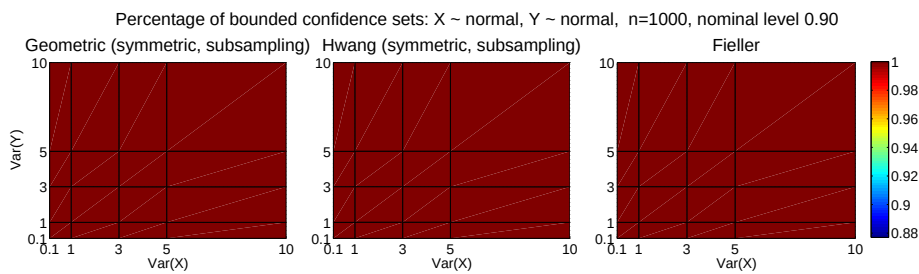
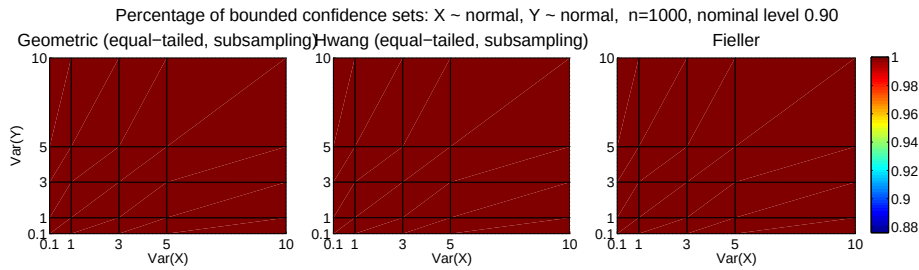
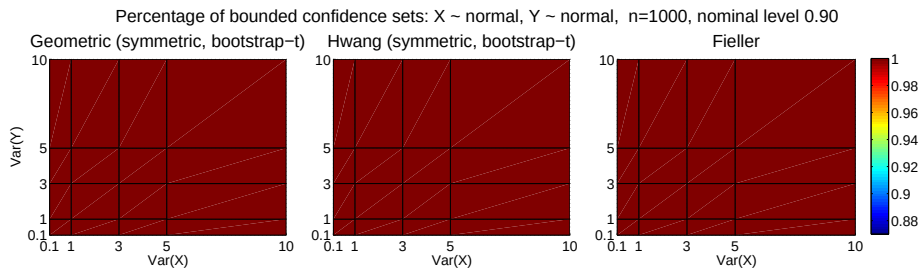
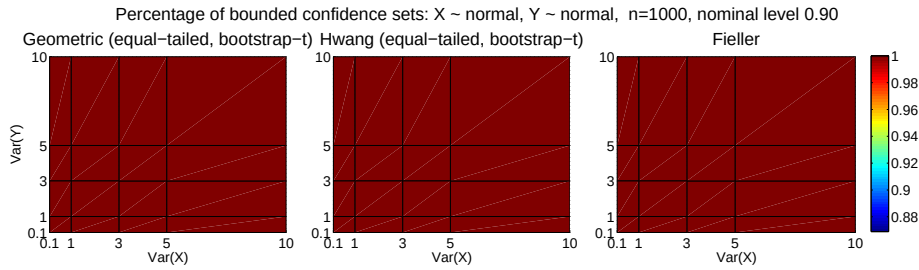
## Number of bounded confidence sets

$X \sim \text{normal}$ ,  $Y \sim \text{normal}$ ,  $n=100$



## Number of bounded confidence sets

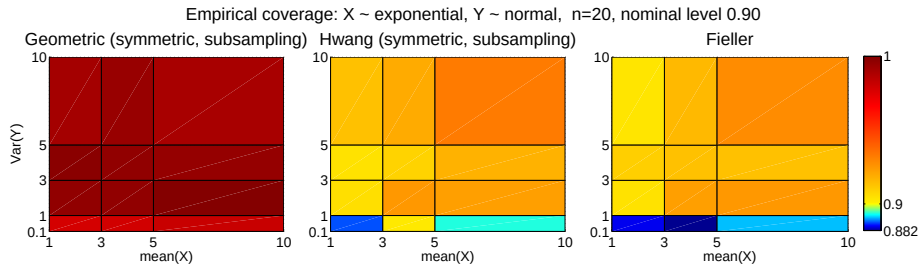
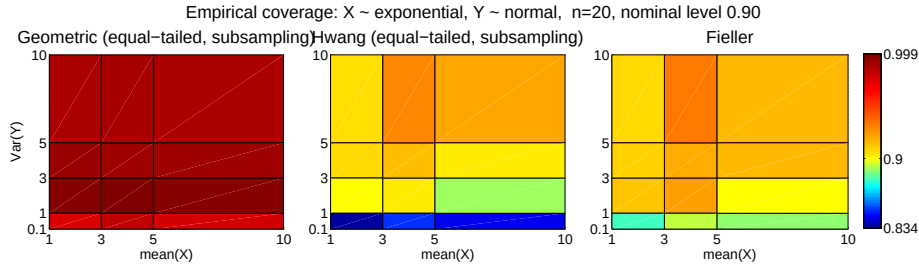
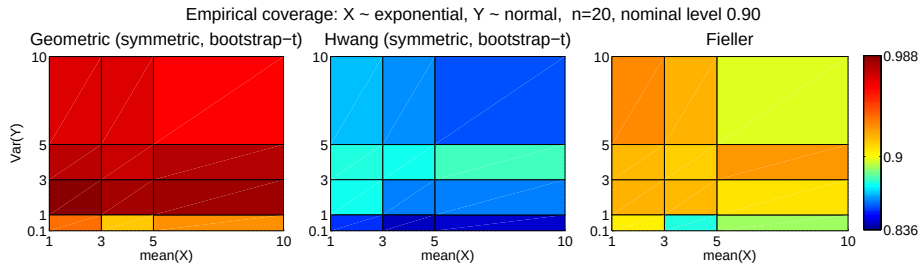
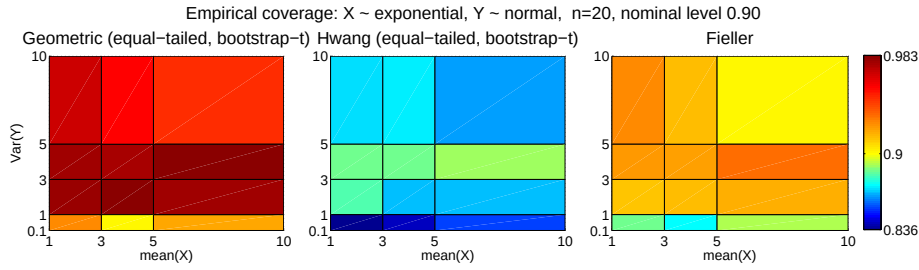
$X \sim \text{normal}$ ,  $Y \sim \text{normal}$ ,  $n=1000$





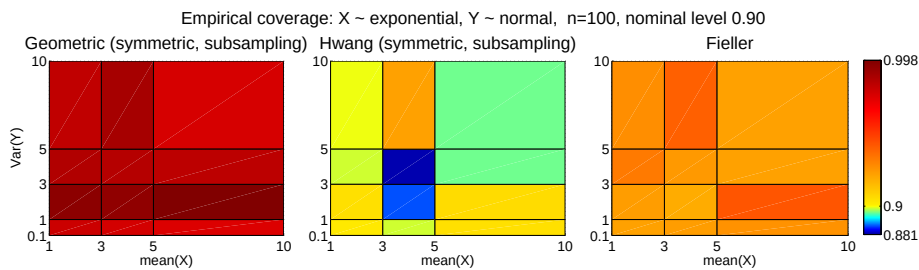
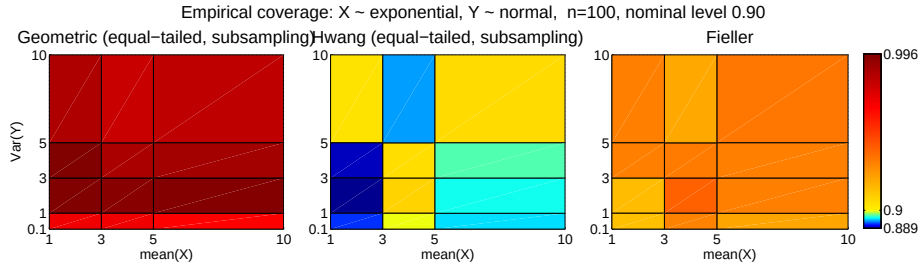
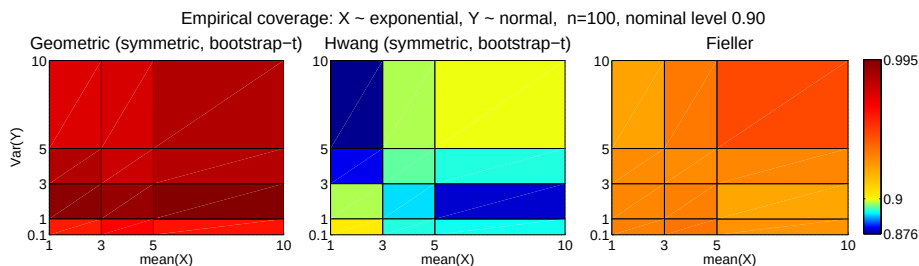
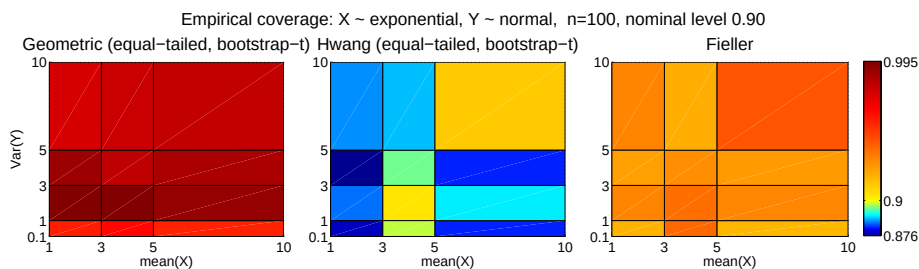
## Empirical coverage

$X \sim \text{exponential}$ ,  $Y \sim \text{normal}$ ,  $n=20$



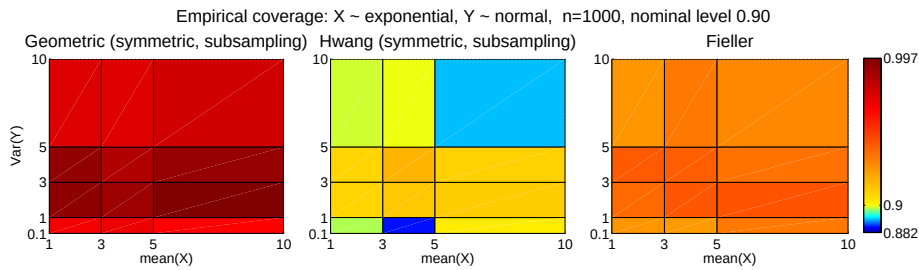
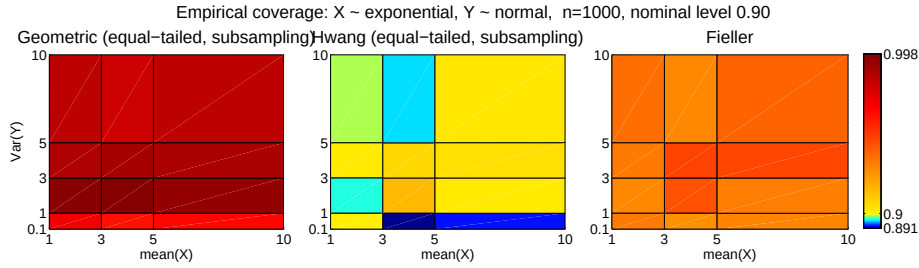
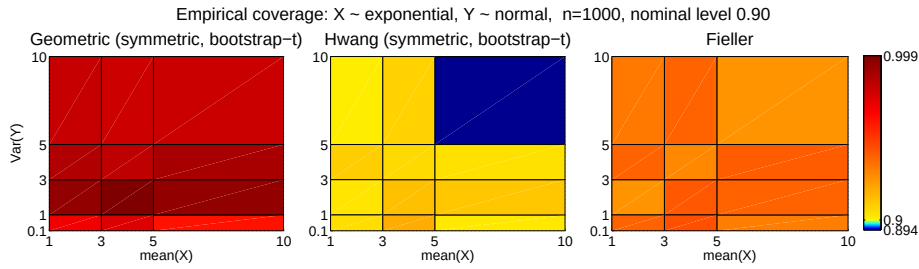
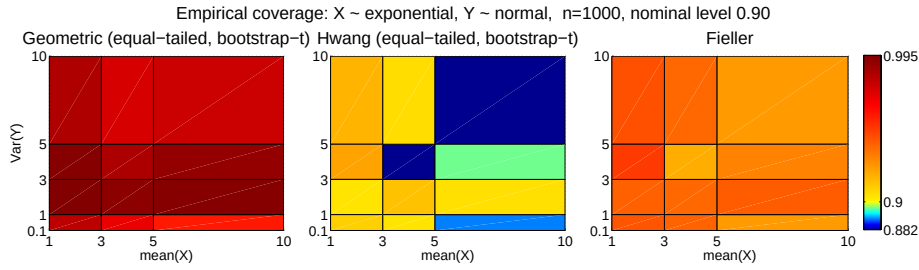
## Empirical coverage

$X \sim \text{exponential}$ ,  $Y \sim \text{normal}$ ,  $n=100$

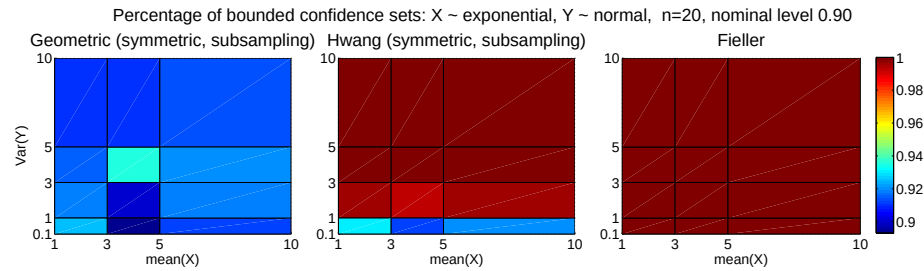
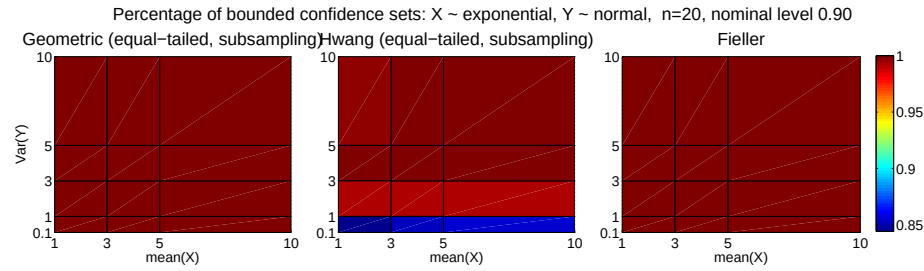
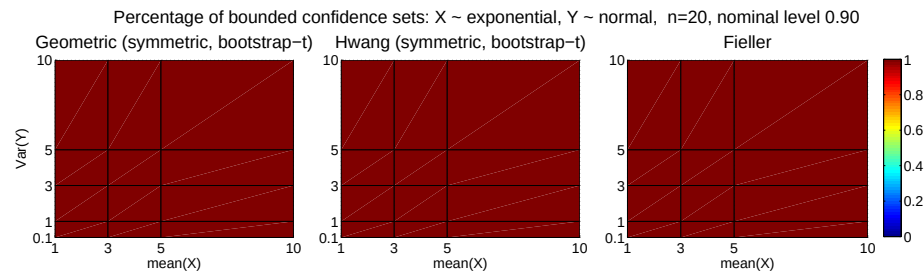
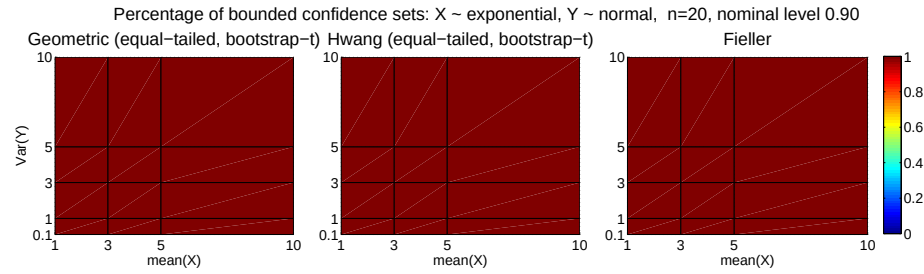


## Empirical coverage

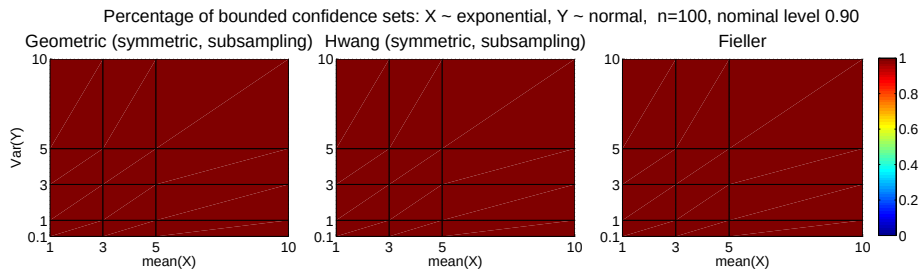
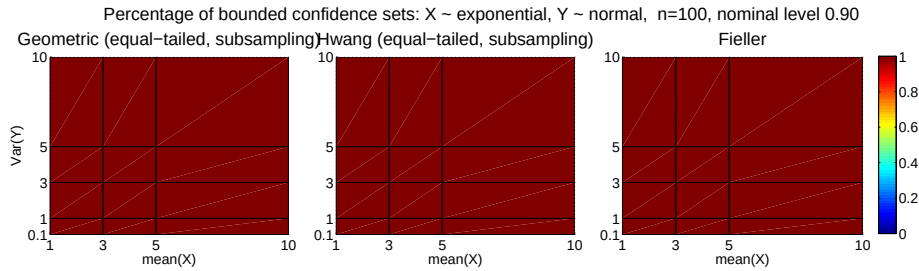
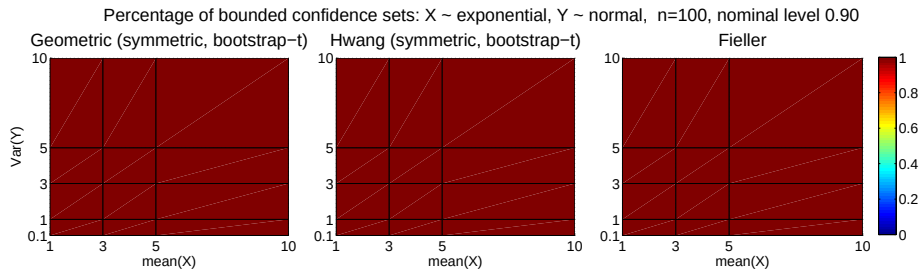
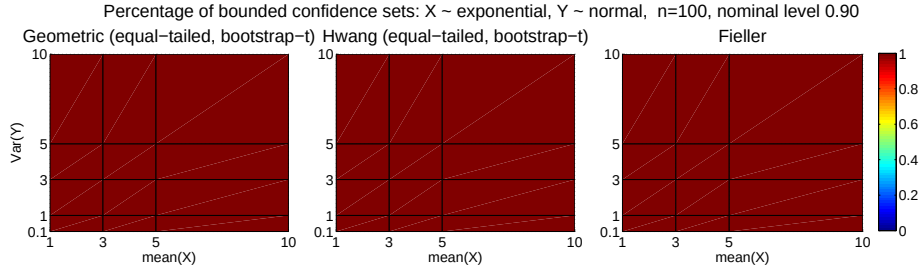
### $X \sim \text{exponential}$ , $Y \sim \text{normal}$ , $n=1000$



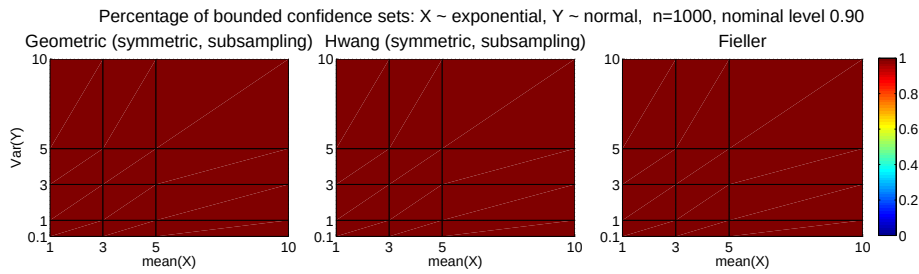
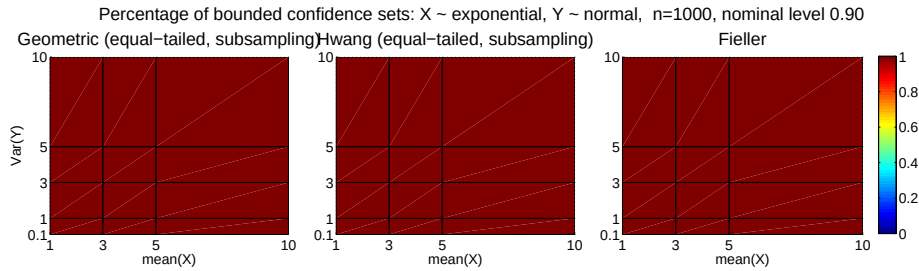
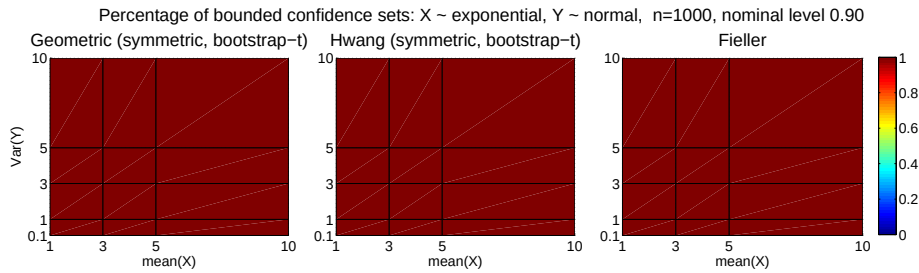
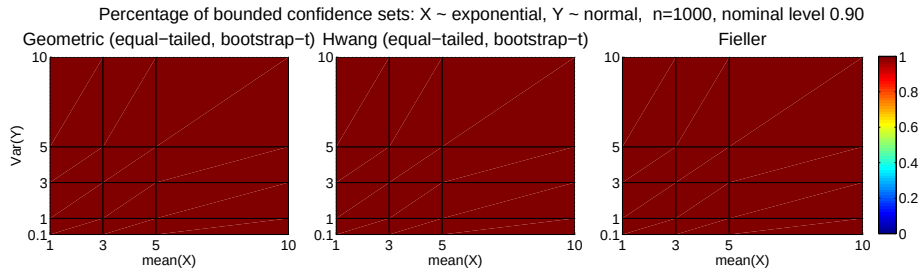
## Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{normal}$ , $n=20$



## Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{normal}$ , $n=100$

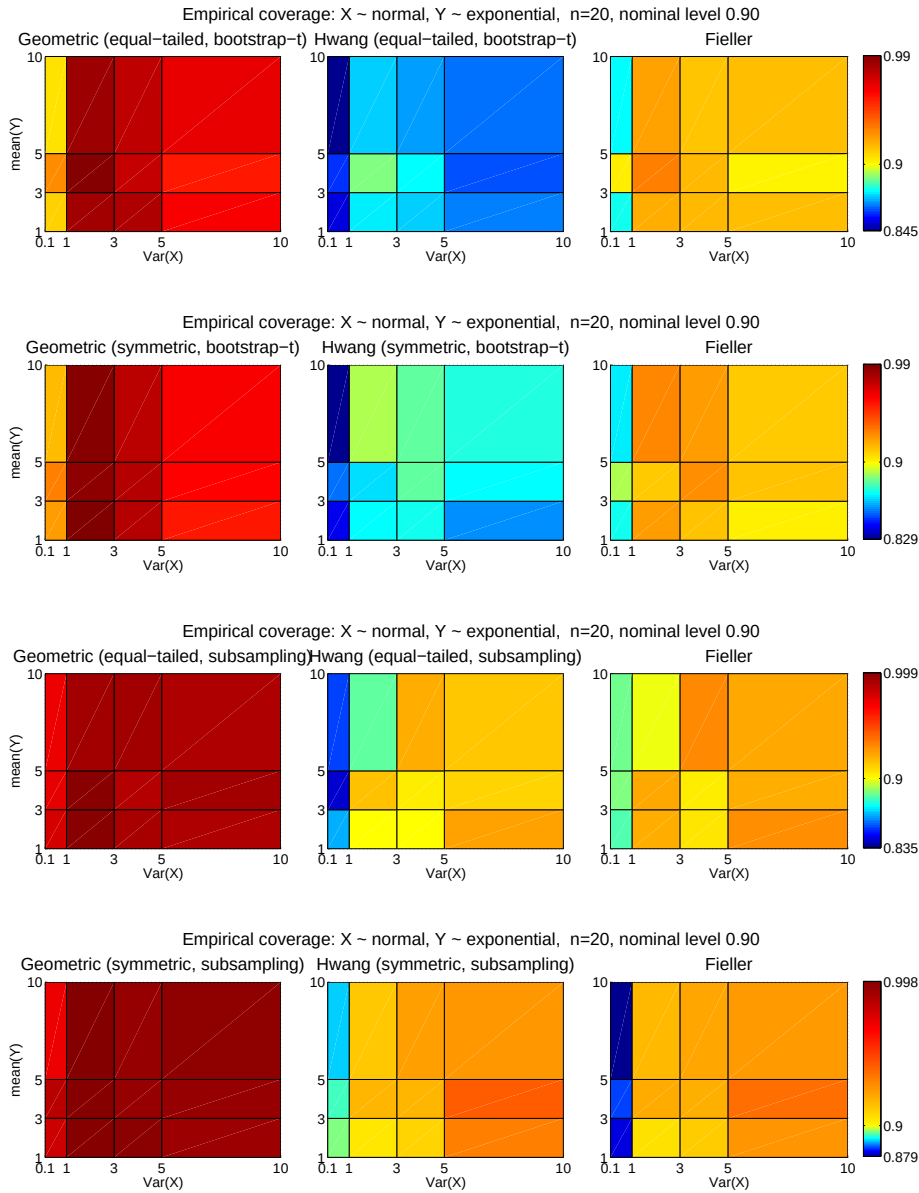


## Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{normal}$ , $n=1000$



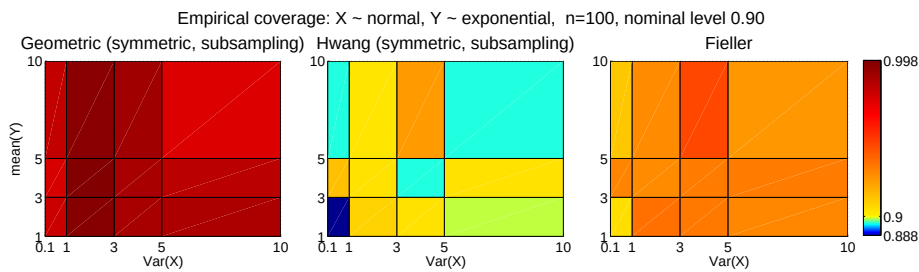
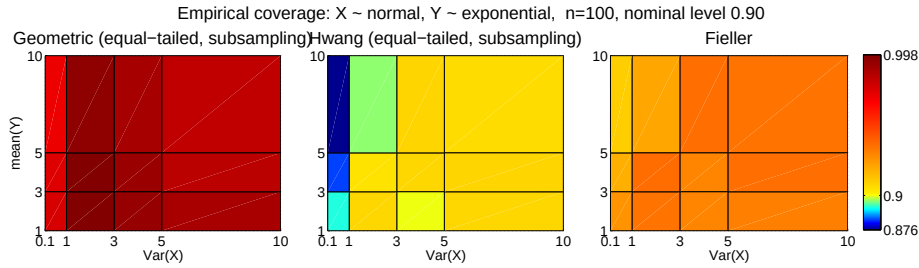
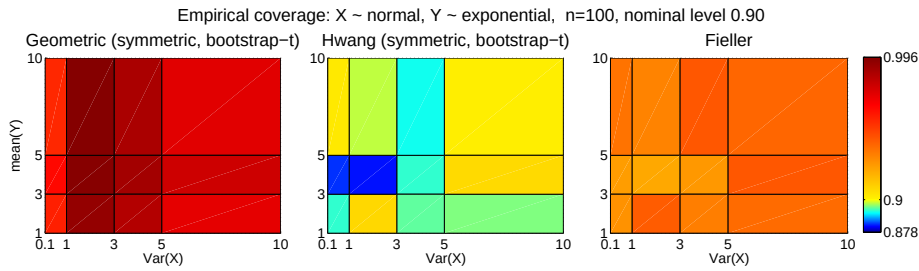
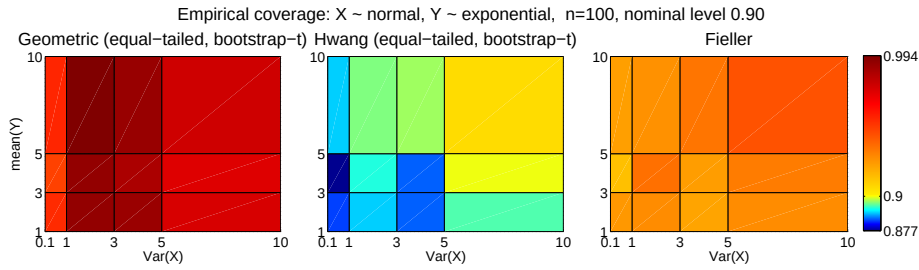
## Empirical coverage

$X \sim \text{normal}$ ,  $Y \sim \text{exponential}$ ,  $n=20$



## Empirical coverage

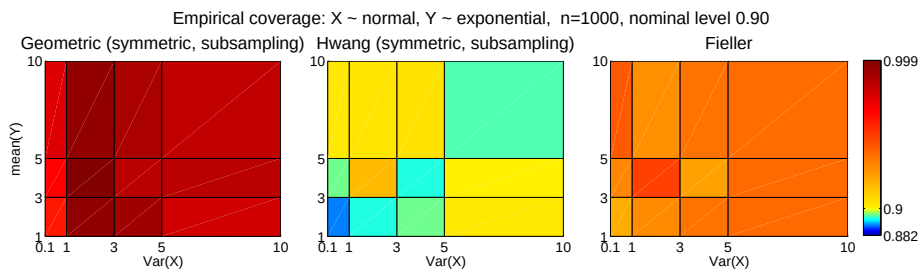
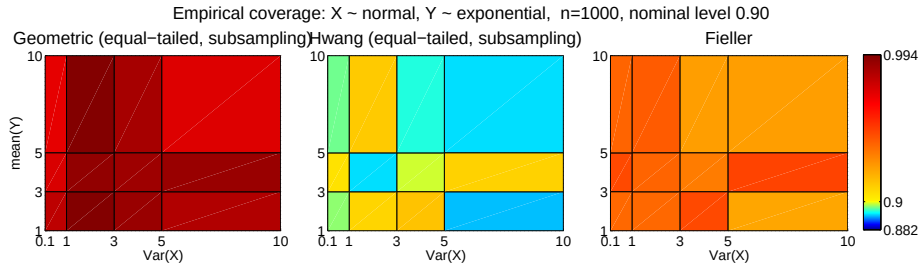
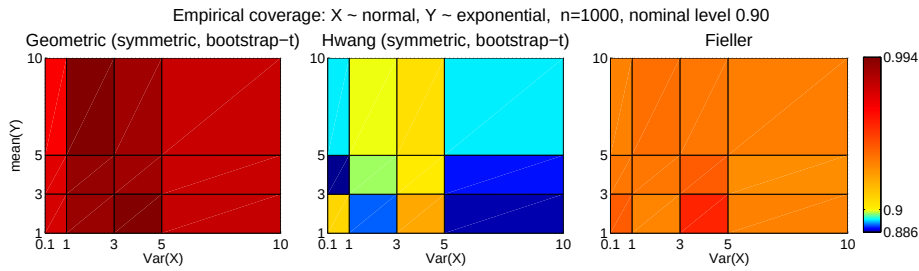
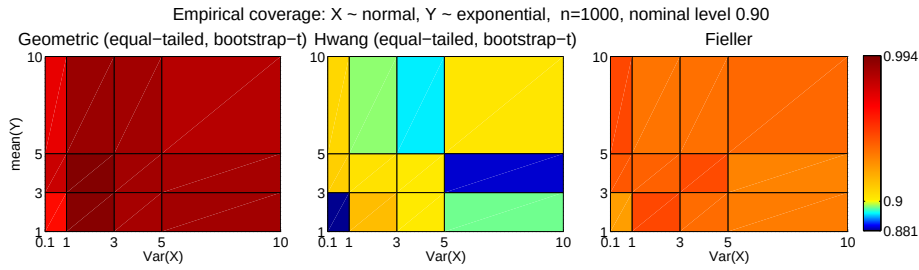
### $X \sim \text{normal}$ , $Y \sim \text{exponential}$ , $n=100$



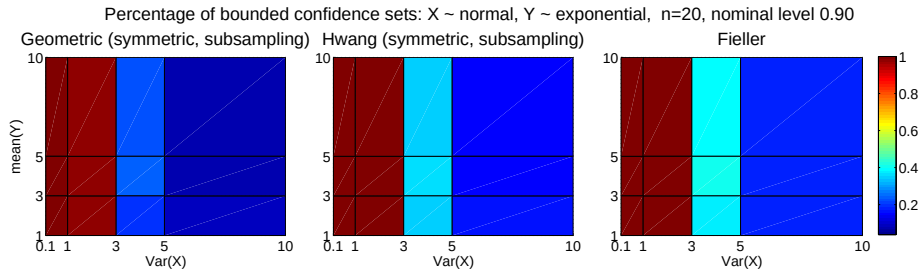
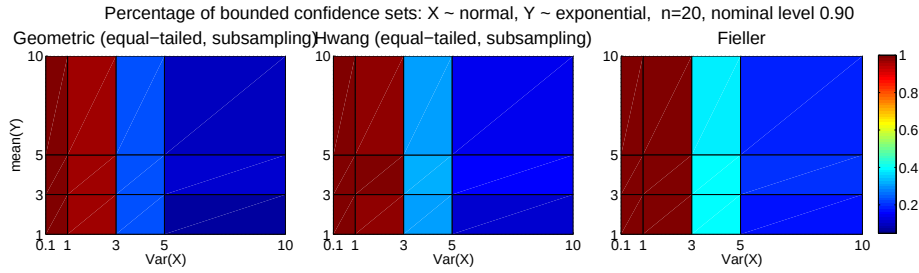
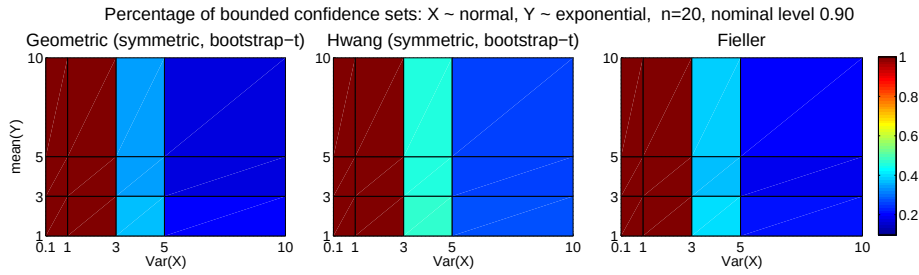
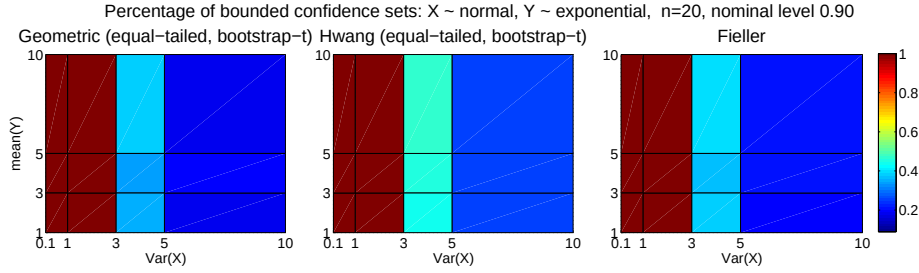


## Empirical coverage

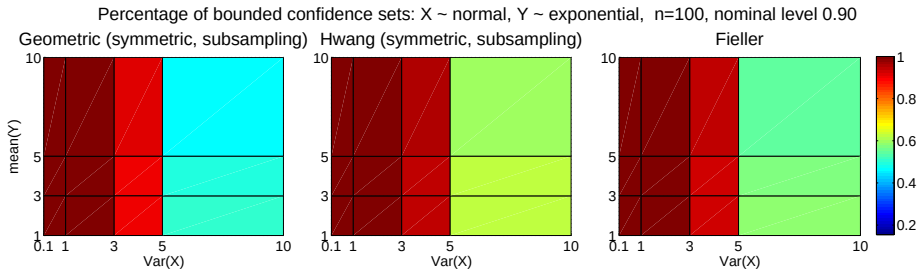
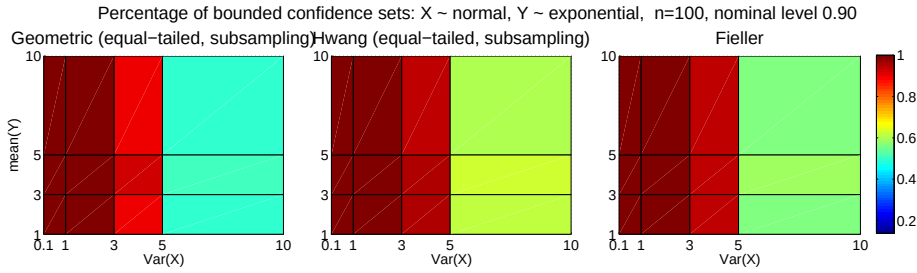
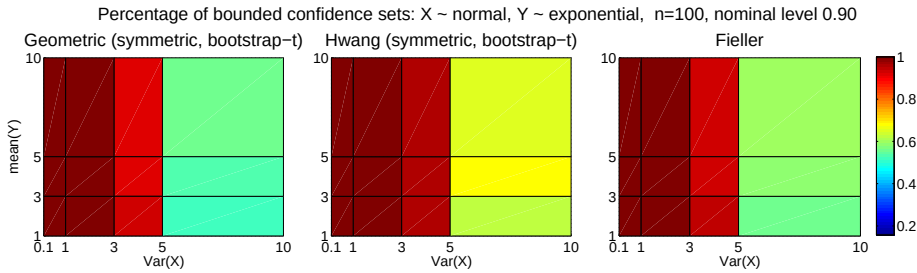
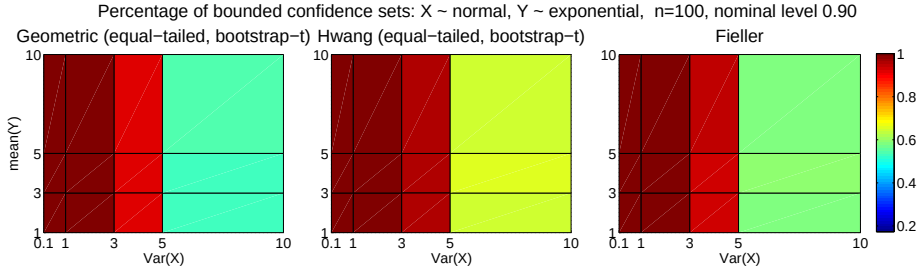
### $X \sim \text{normal}$ , $Y \sim \text{exponential}$ , $n=1000$



## Number of bounded confidence sets $X \sim \text{normal}$ , $Y \sim \text{exponential}$ , $n=20$

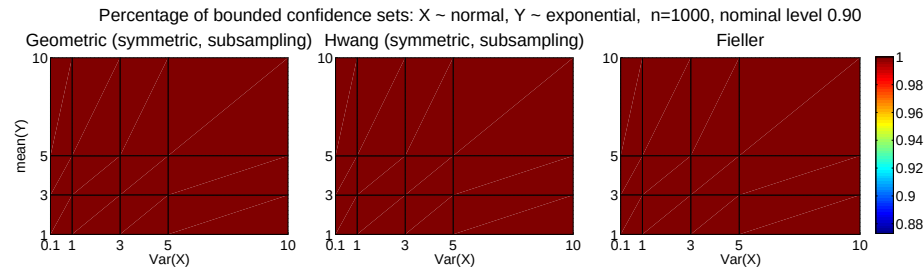
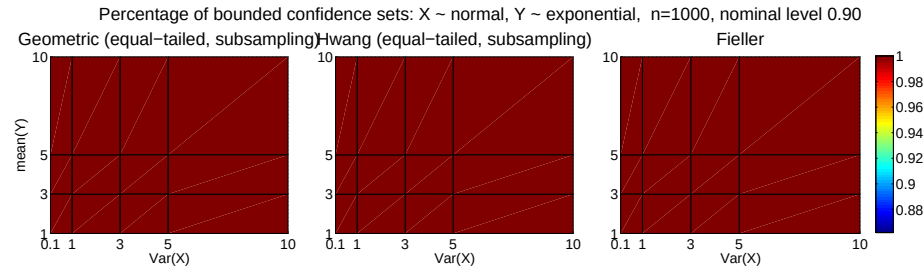
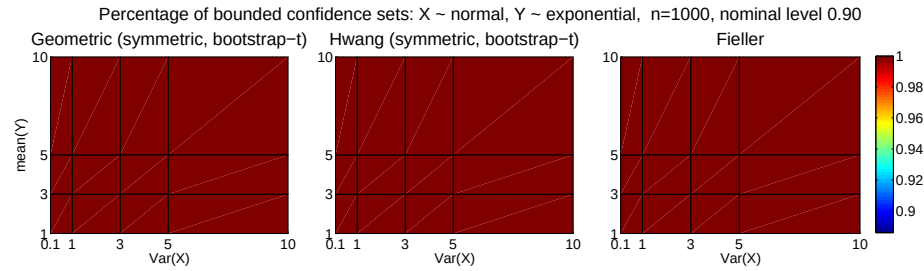
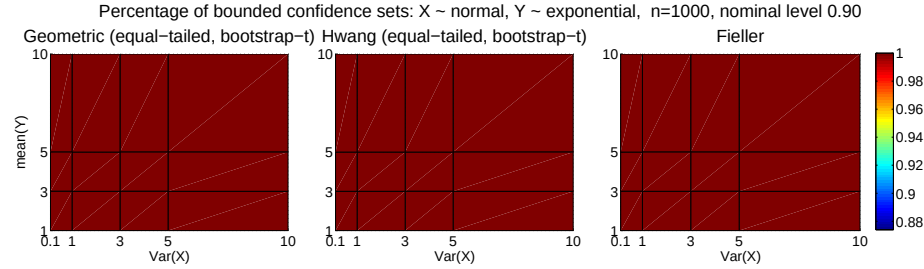


## Number of bounded confidence sets $X \sim \text{normal}$ , $Y \sim \text{exponential}$ , $n=100$

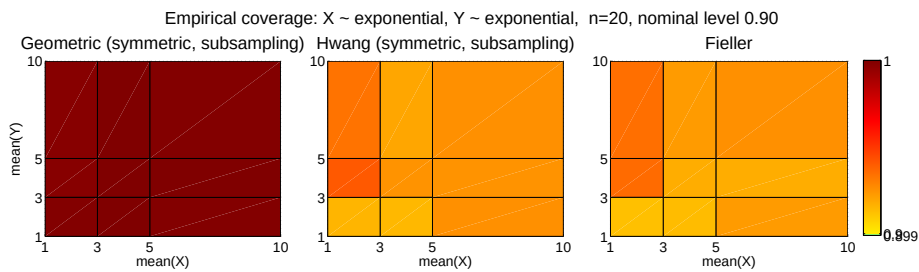
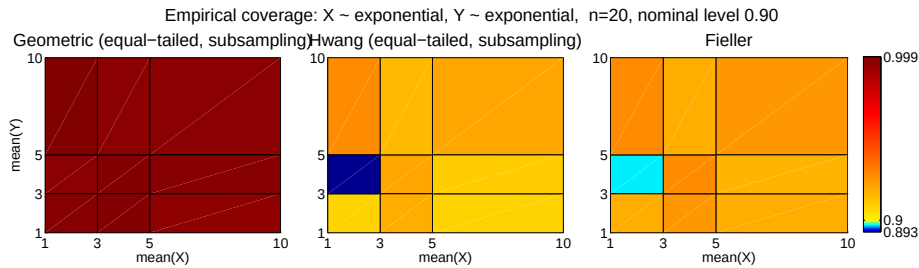
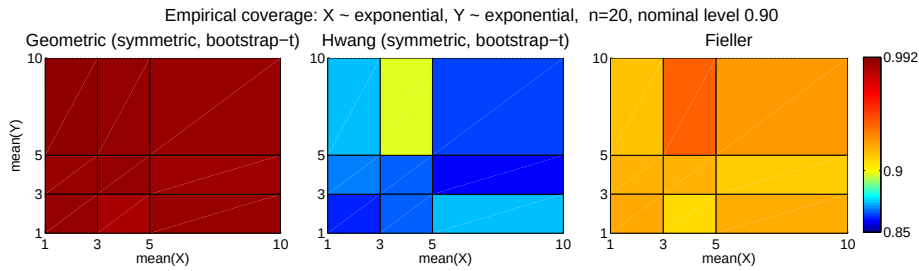
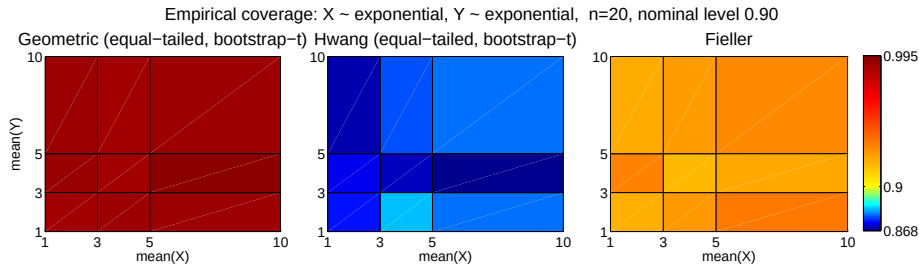


## Number of bounded confidence sets

$X \sim \text{normal}$ ,  $Y \sim \text{exponential}$ ,  $n=1000$

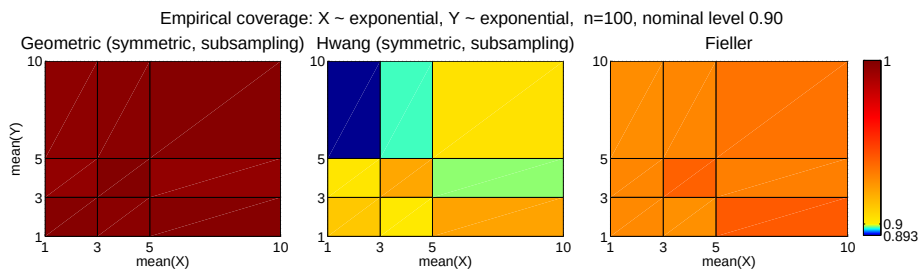
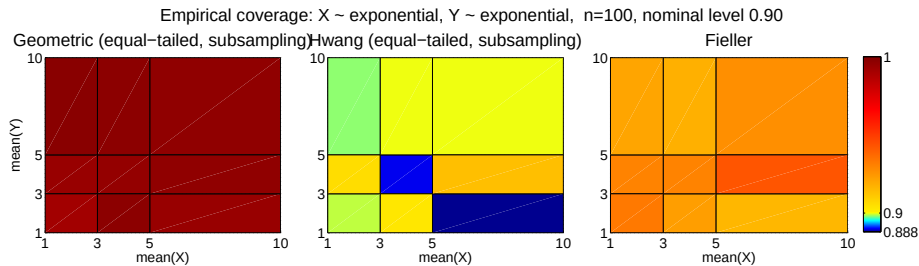
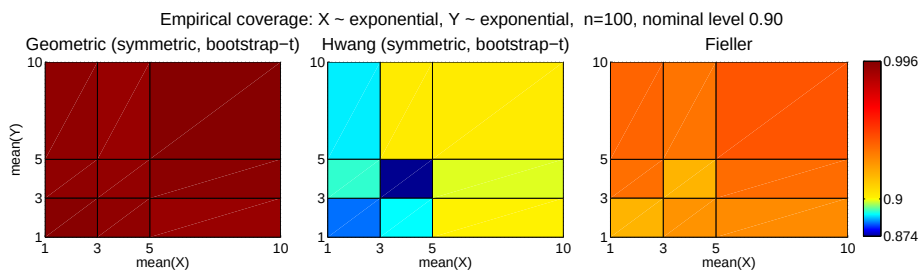
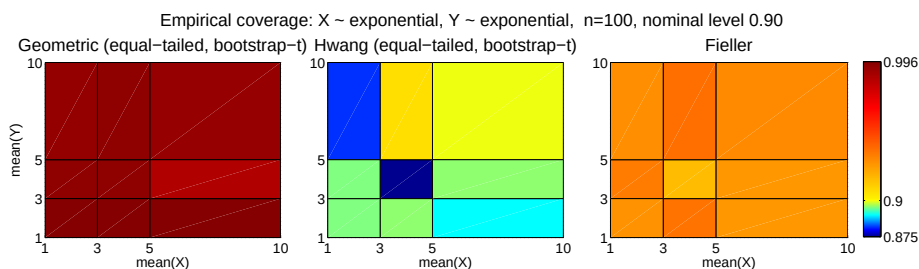


## Empirical coverage $X \sim \text{exponential}$ , $Y \sim \text{exponential}$ , $n=20$

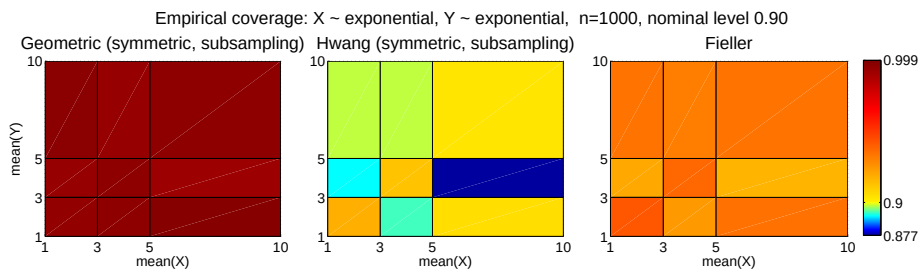
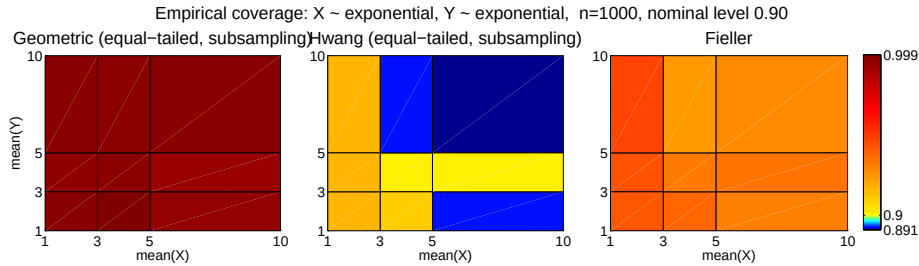
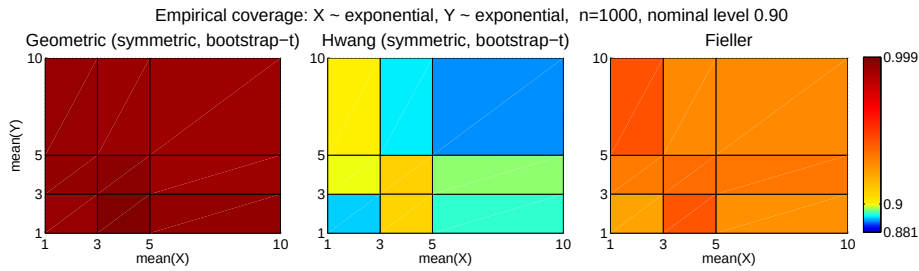
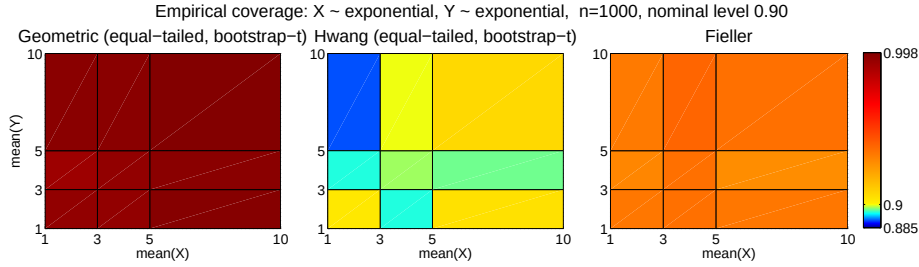


## Empirical coverage

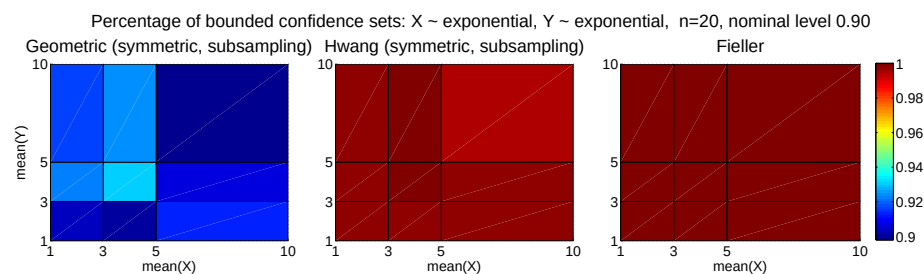
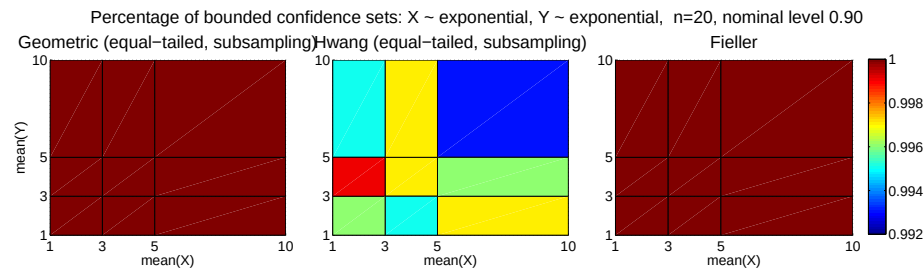
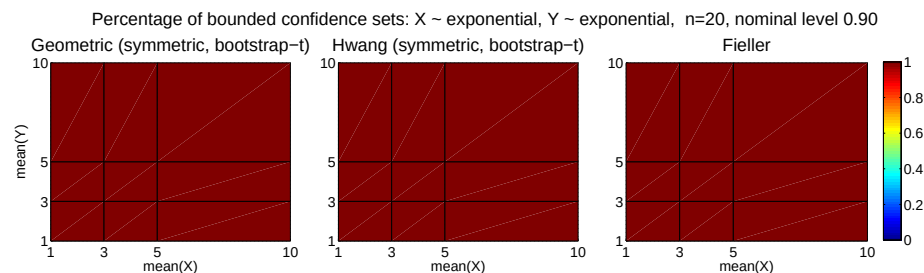
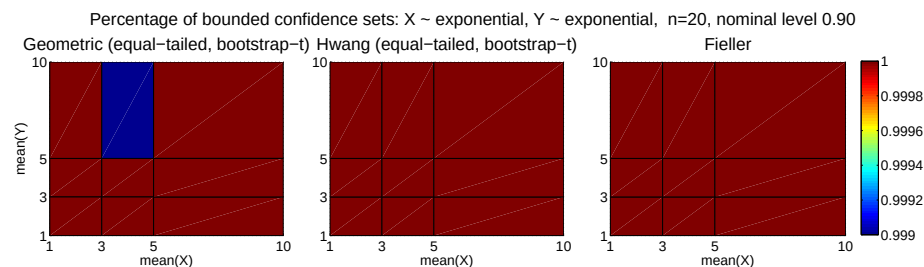
### $X \sim \text{exponential}$ , $Y \sim \text{exponential}$ , $n=100$



# Empirical coverage $X \sim \text{exponential}$ , $Y \sim \text{exponential}$ , $n=1000$

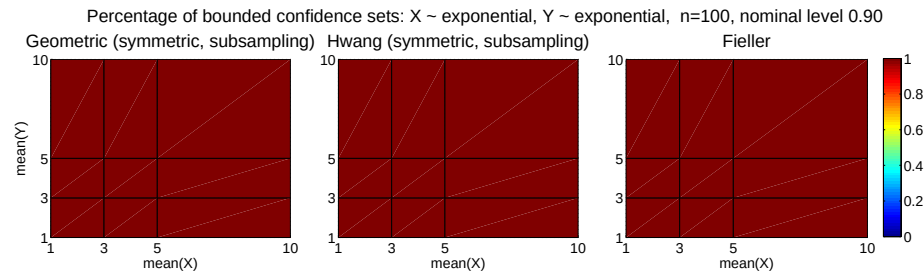
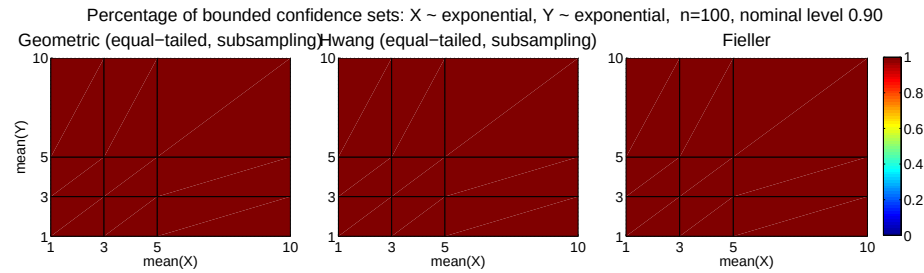
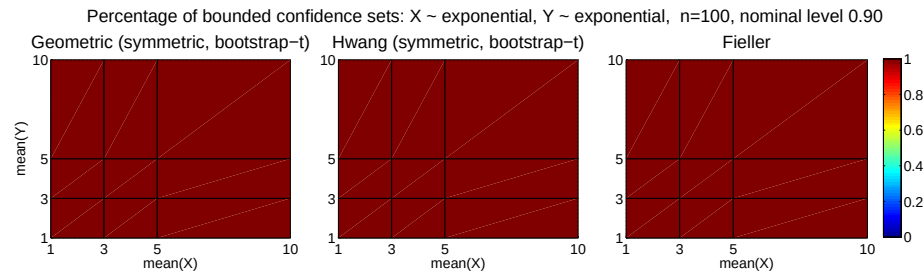
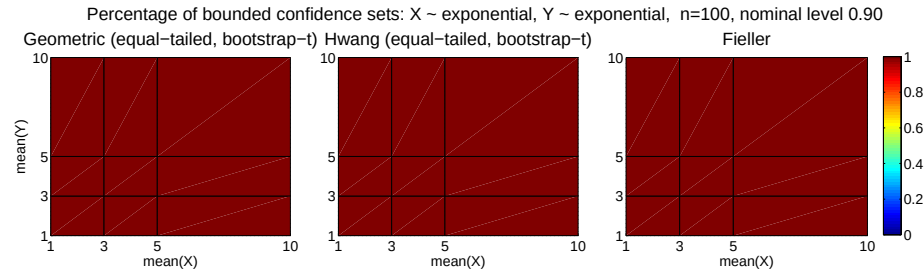


## Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{exponential}$ , $n=20$

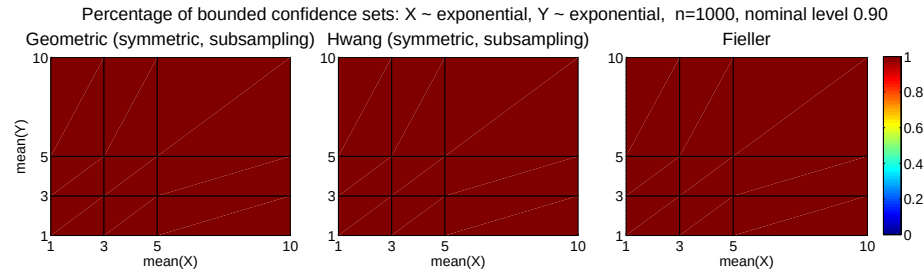
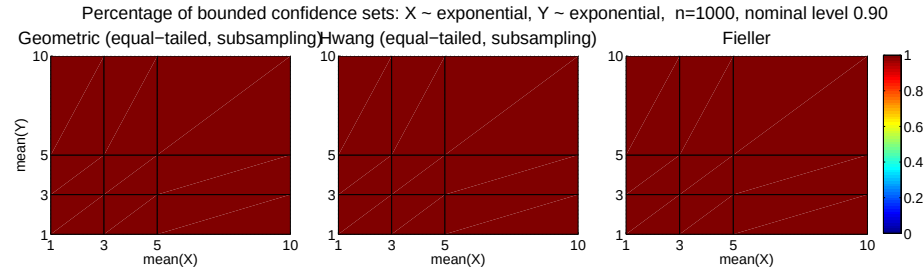
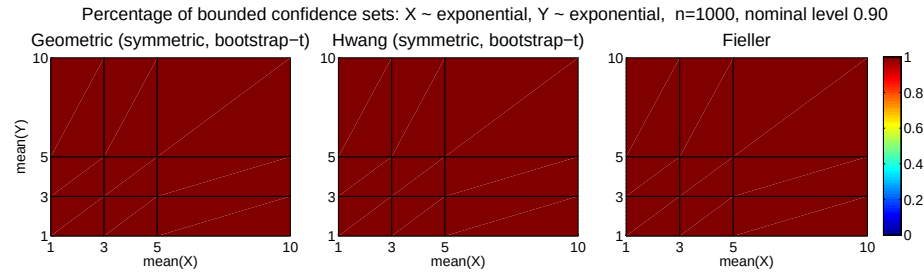
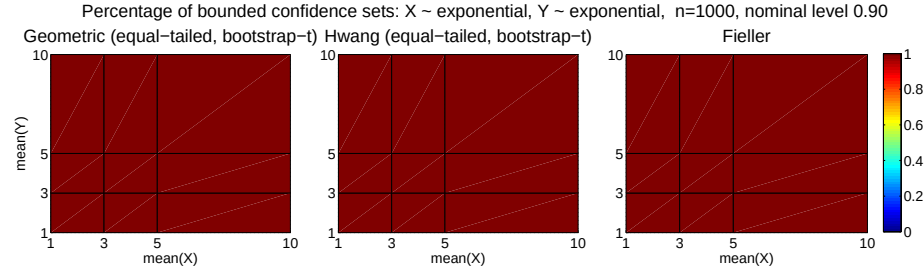




## Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{exponential}$ , $n=100$

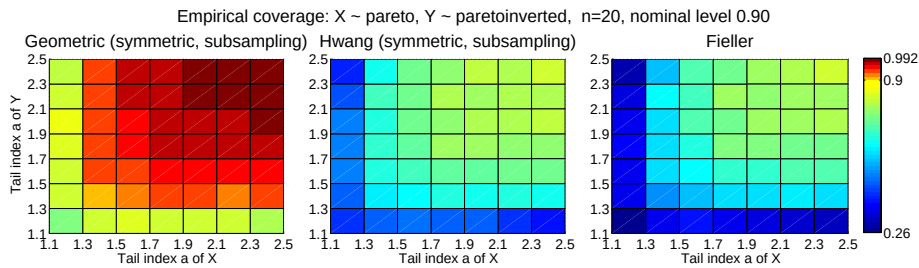
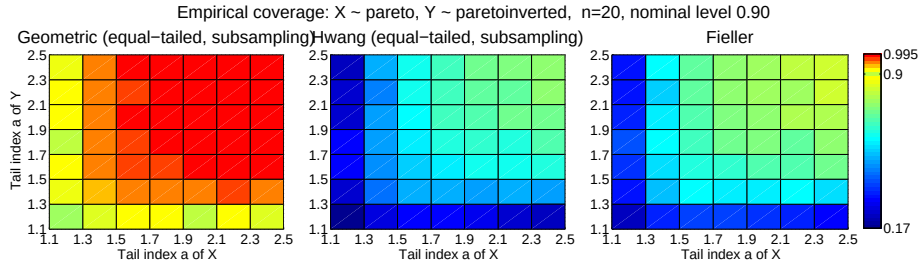
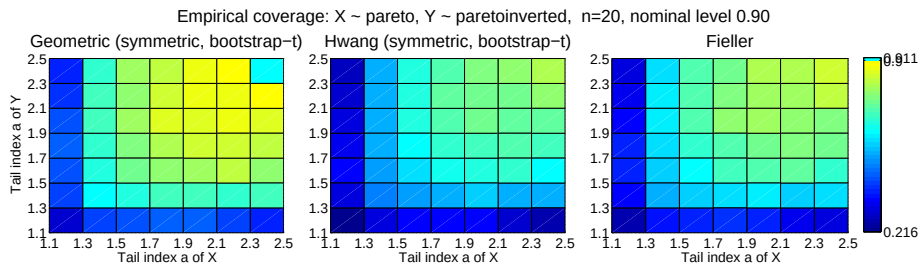
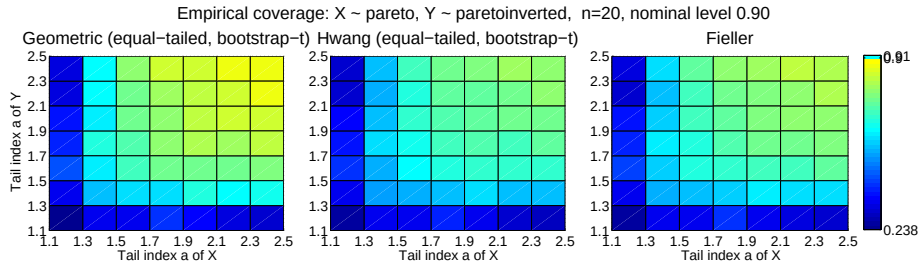


## Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{exponential}$ , $n=1000$



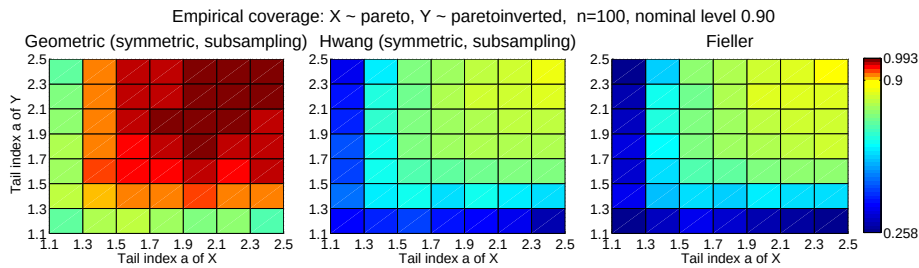
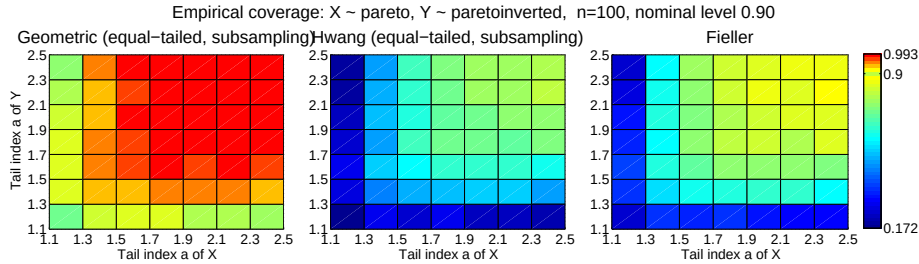
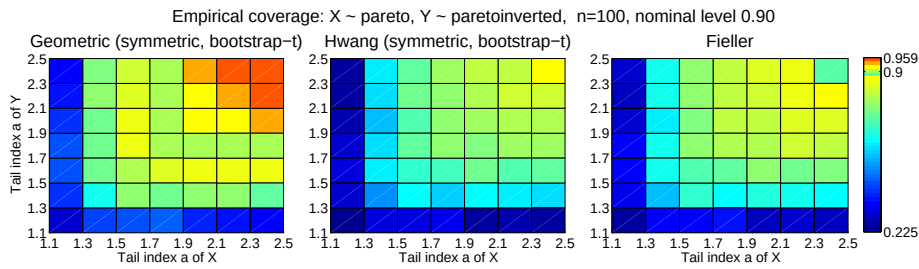
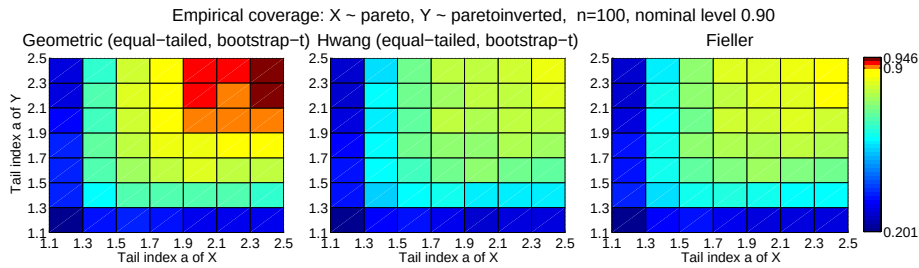
## Empirical coverage

### $X \sim \text{Pareto}, Y \sim \text{Paretoinverted}, n=20$



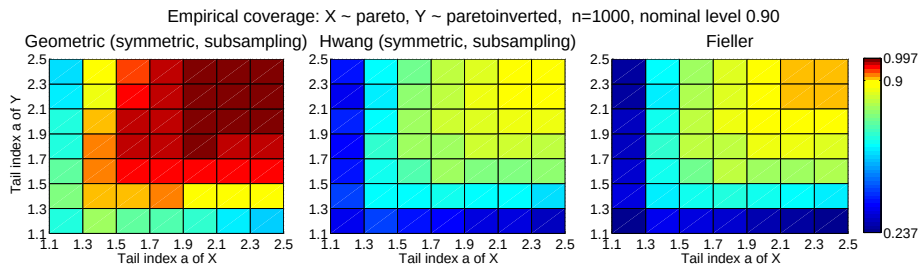
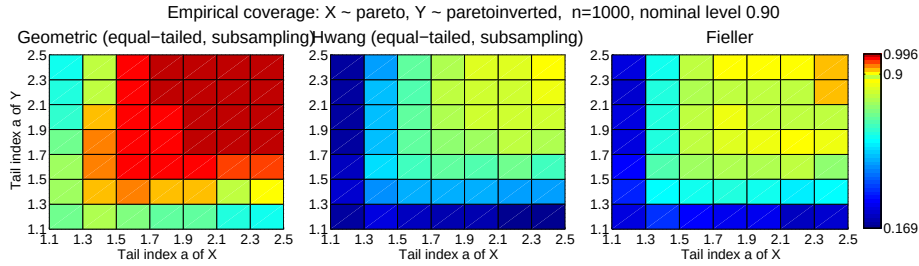
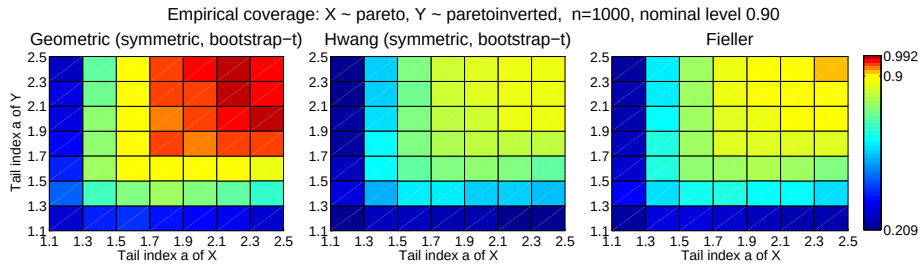
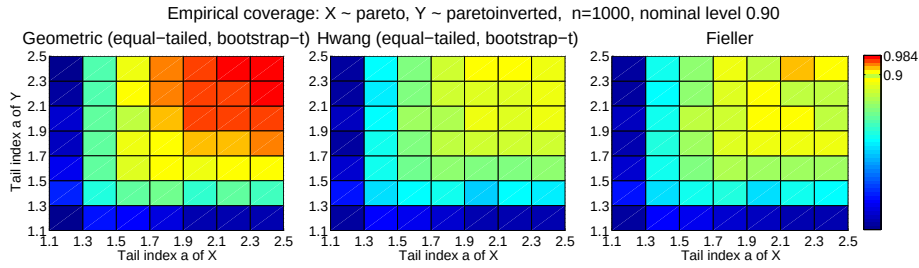
## Empirical coverage

### $X \sim \text{Pareto}$ , $Y \sim \text{Paretoinv}$ , $n=100$

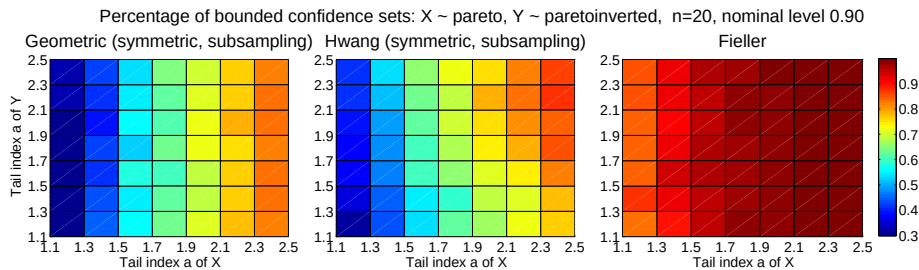
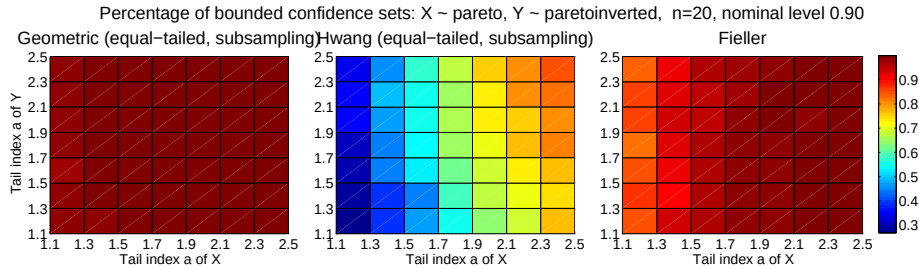
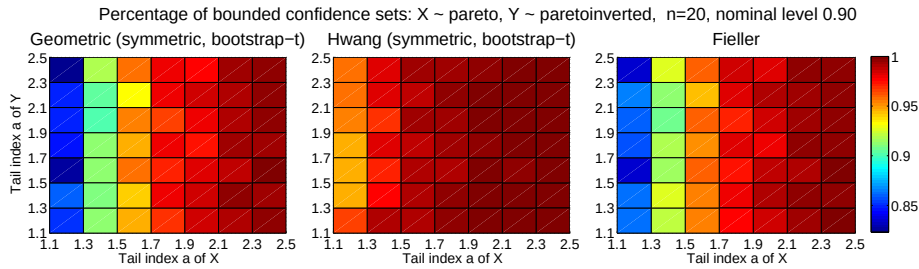
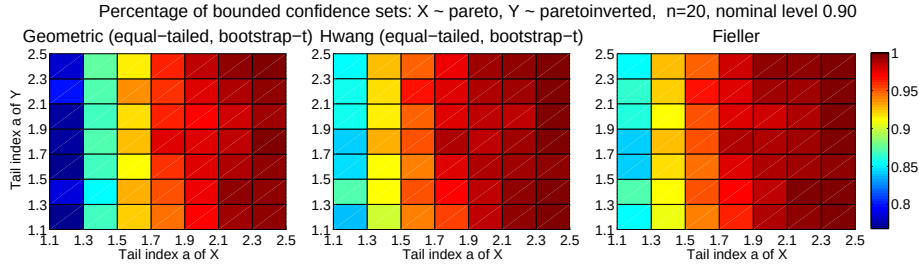


## Empirical coverage

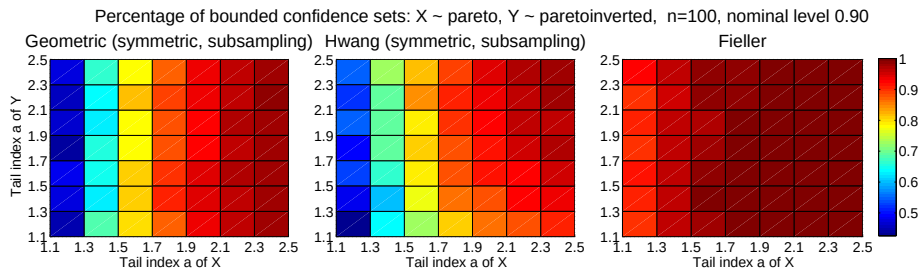
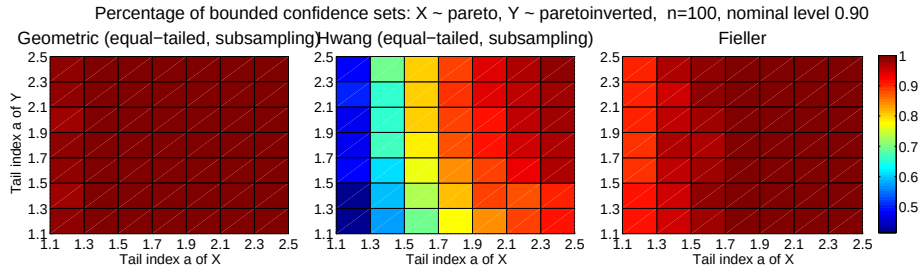
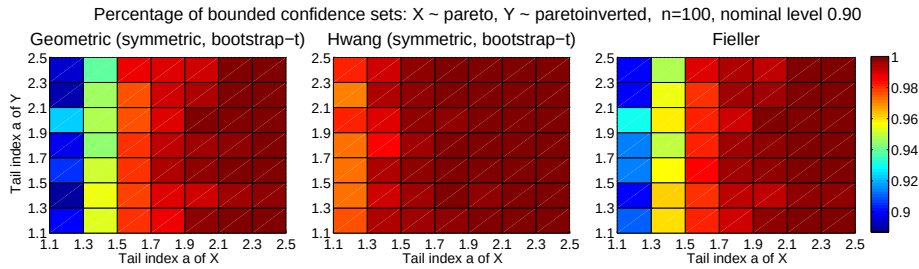
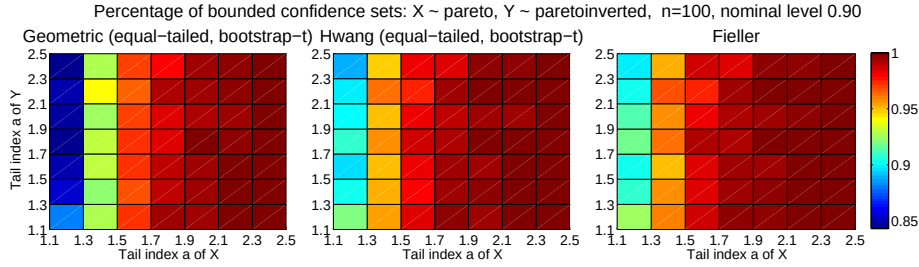
### $X \sim \text{Pareto}, Y \sim \text{Paretoinverted}, n=1000$



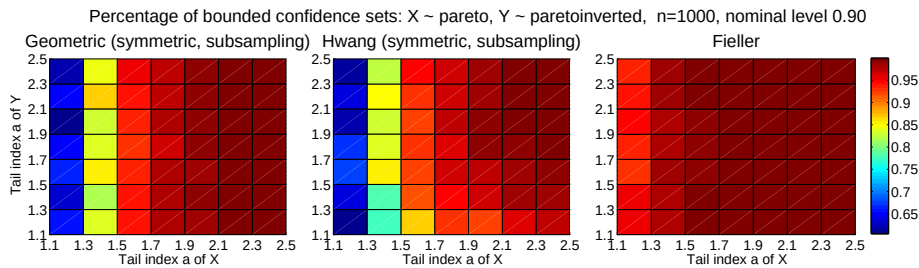
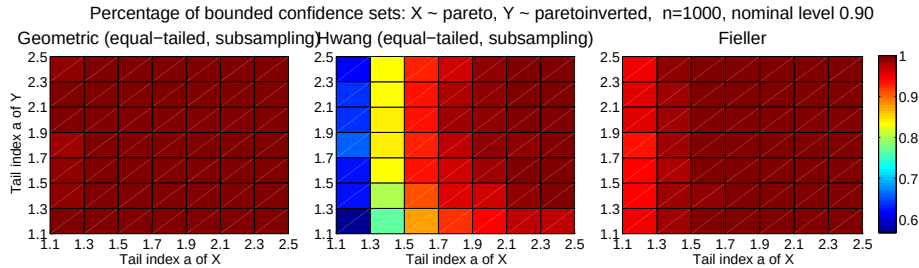
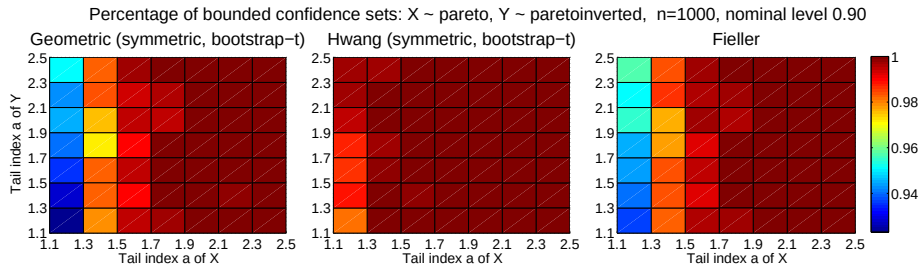
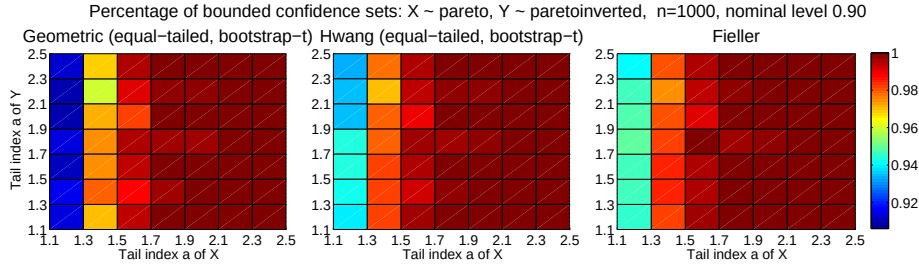
## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{Paretoinverted}$ , $n=20$



## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{Paretoinverted}$ , $n=100$

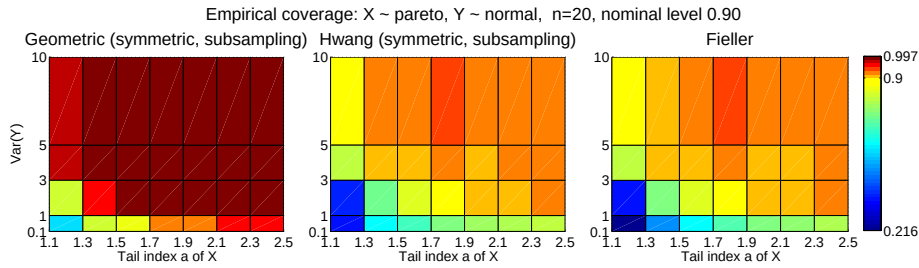
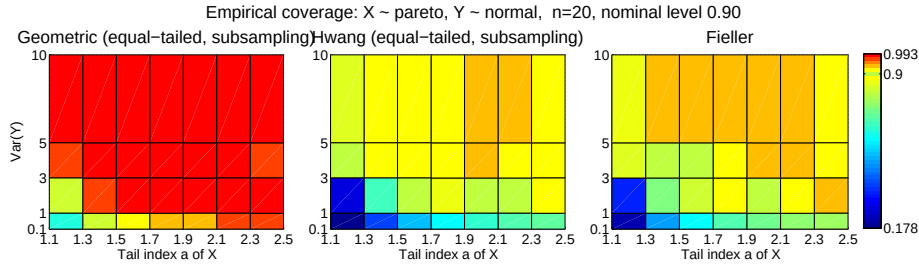
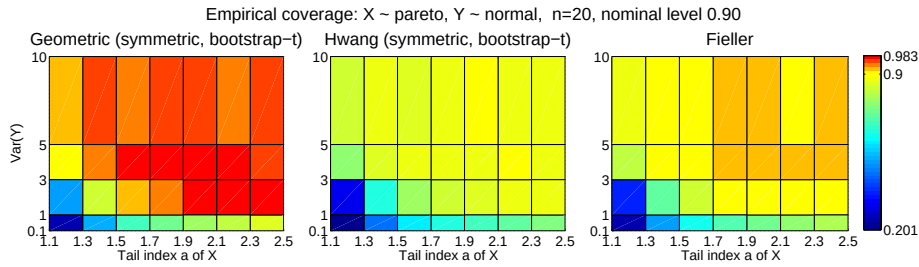
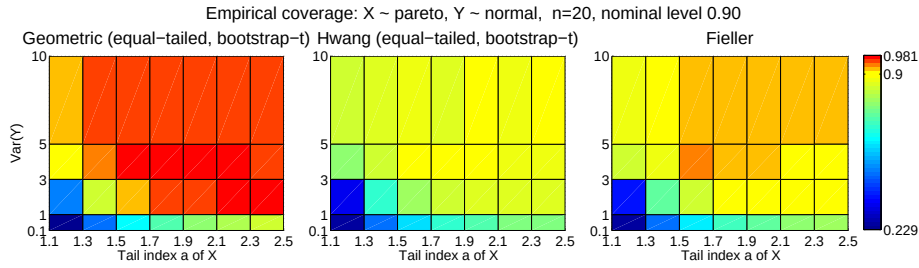


## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{Paretoinv}$ , $n=1000$



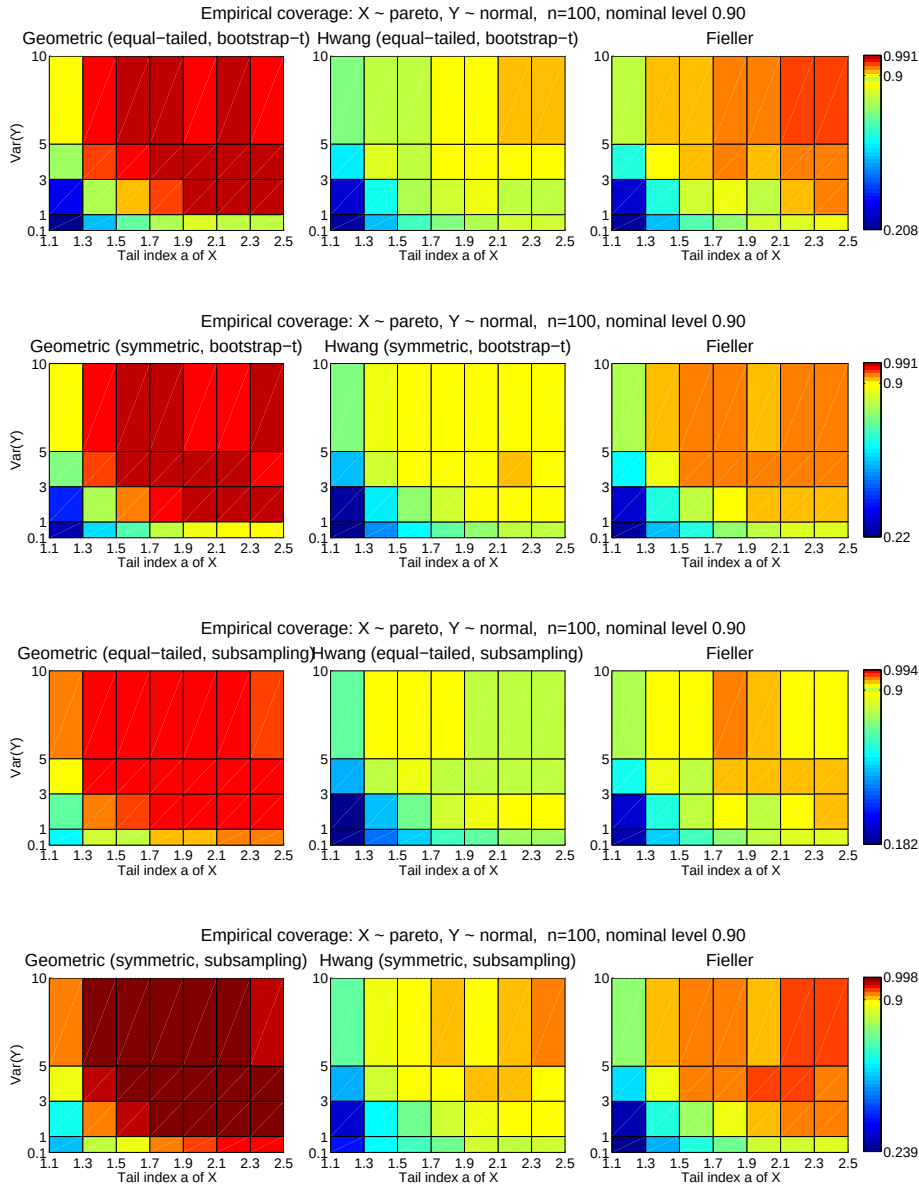


# Empirical coverage $X \sim \text{Pareto}$ , $Y \sim \text{normal}$ , $n=20$



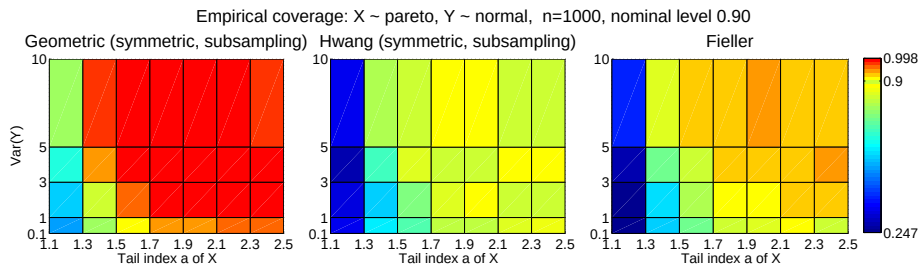
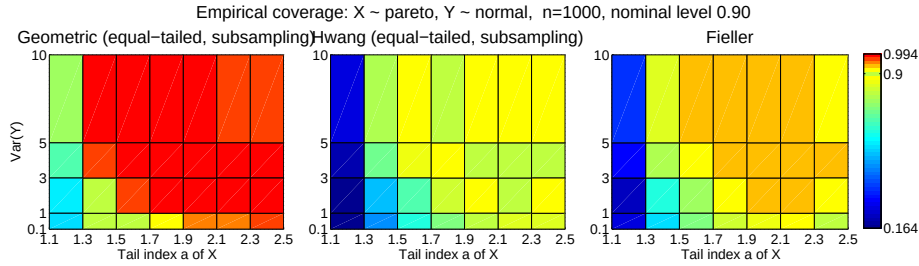
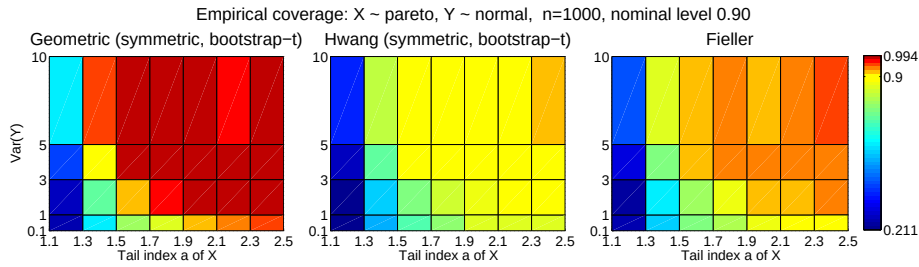
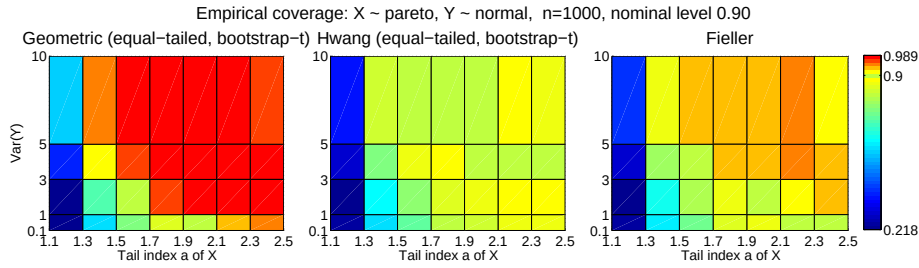
## Empirical coverage

### $X \sim \text{Pareto}$ , $Y \sim \text{normal}$ , $n=100$



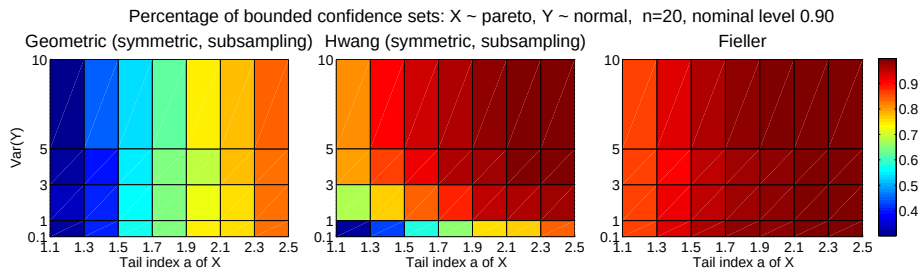
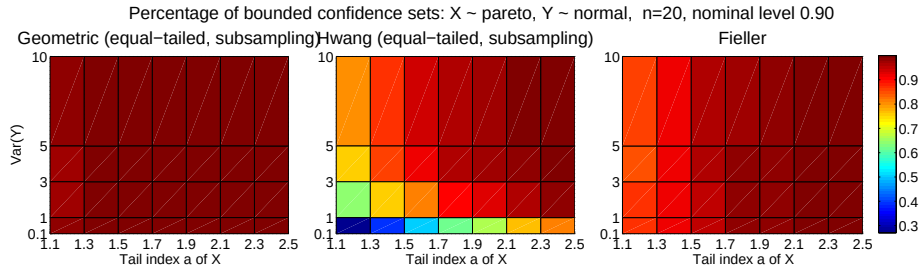
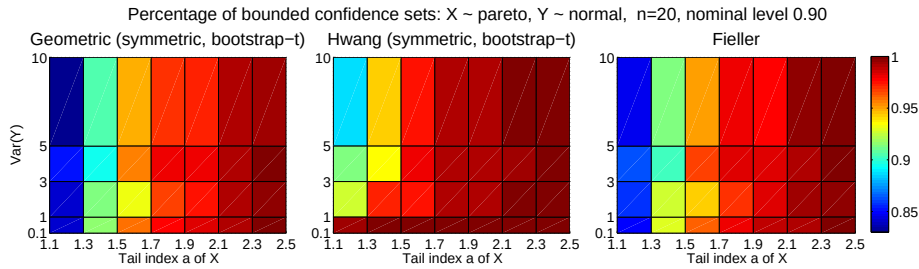
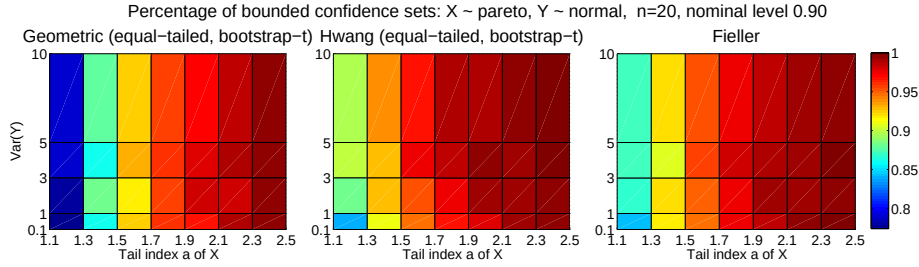
## Empirical coverage

### $X \sim \text{Pareto}$ , $Y \sim \text{normal}$ , $n=1000$

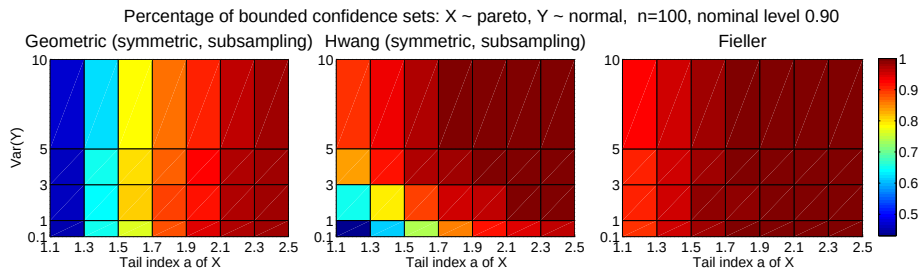
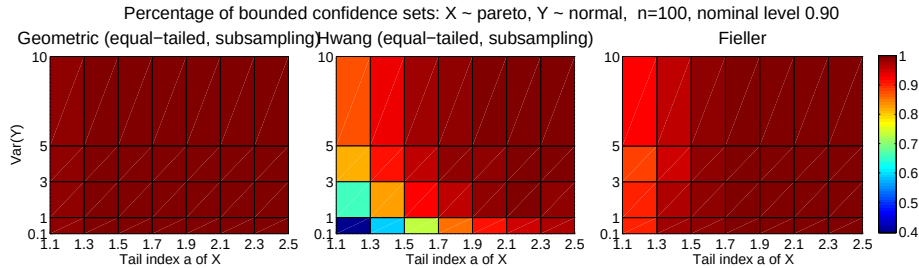
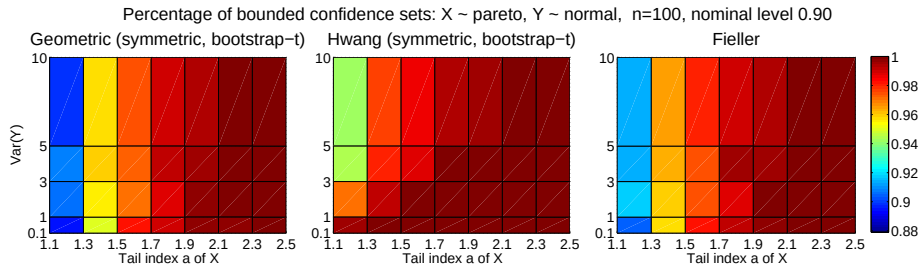
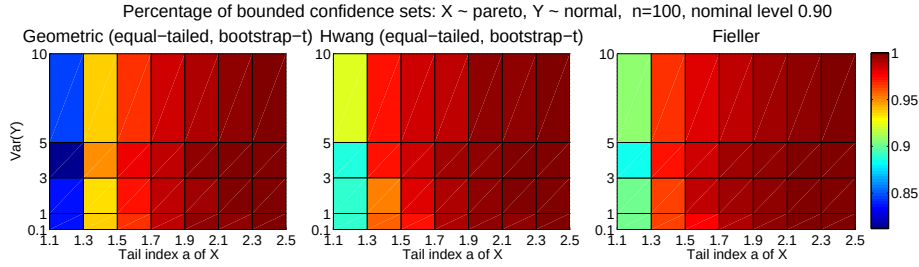


## Number of bounded confidence sets

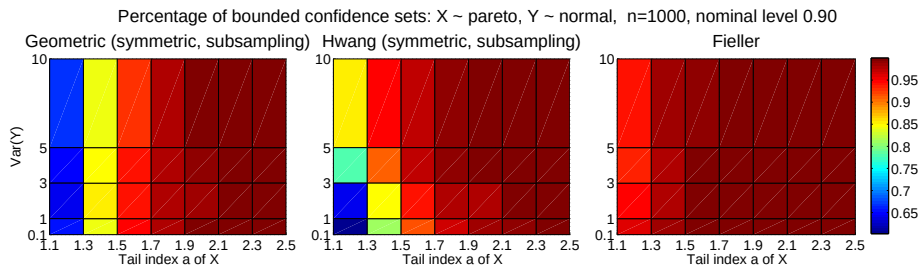
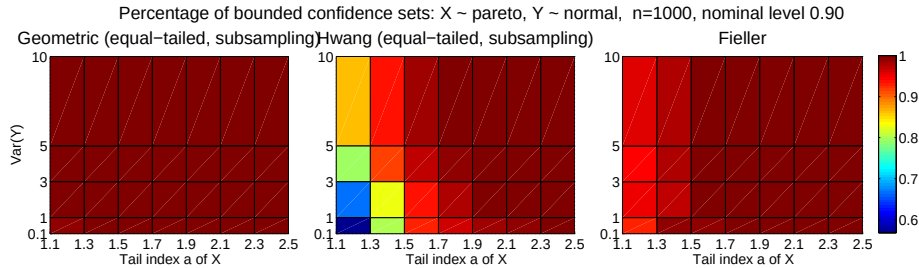
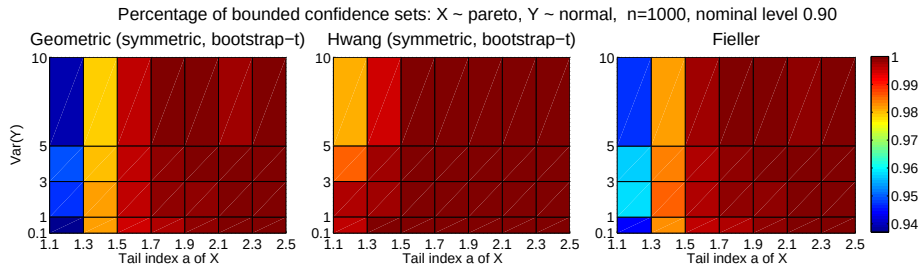
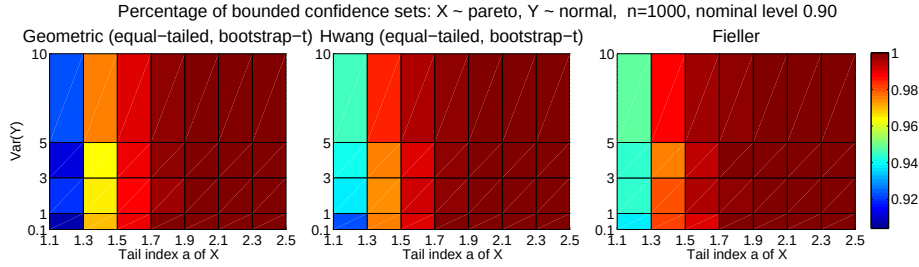
$X \sim \text{Pareto}$ ,  $Y \sim \text{normal}$ ,  $n=20$



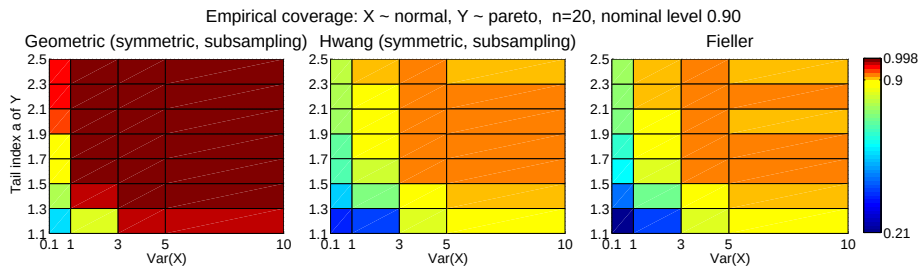
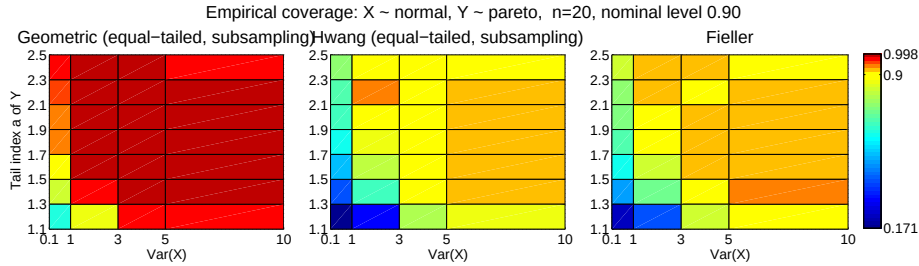
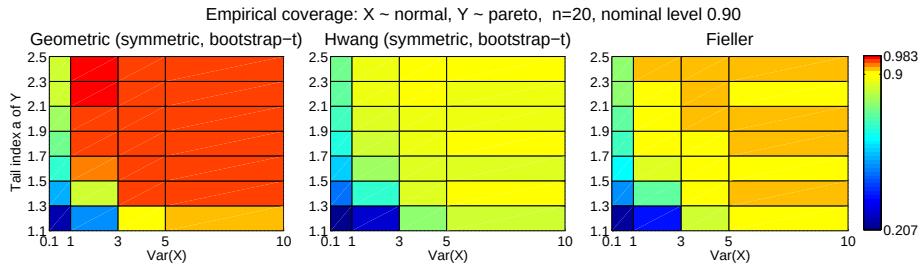
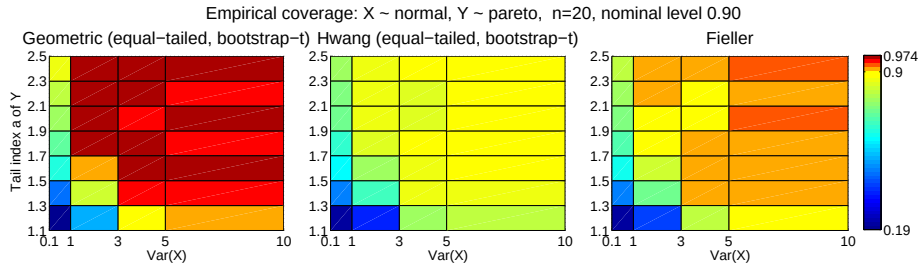
## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{normal}$ , $n=100$



## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{normal}$ , $n=1000$

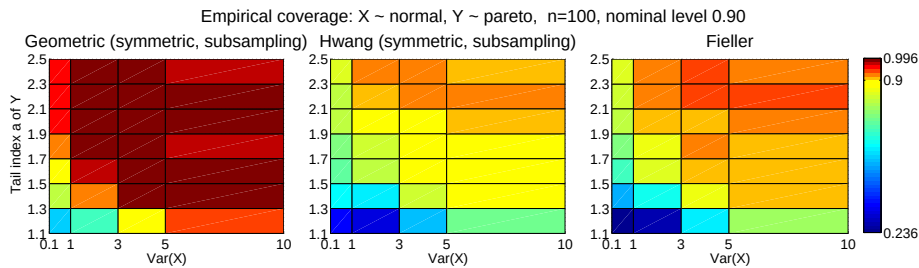
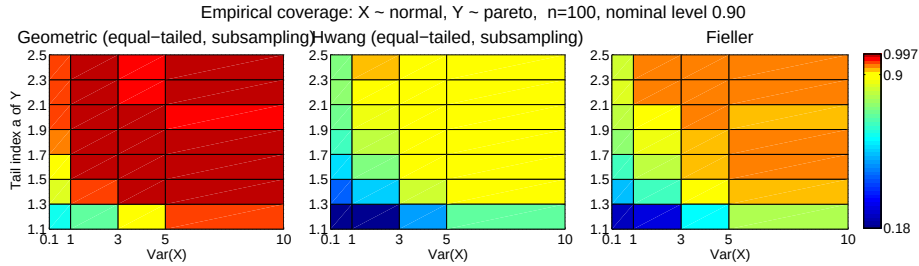
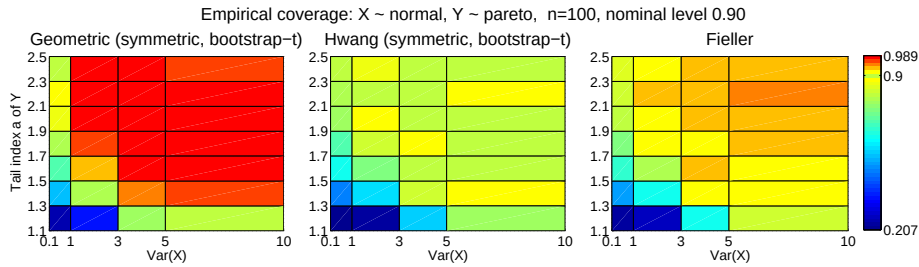
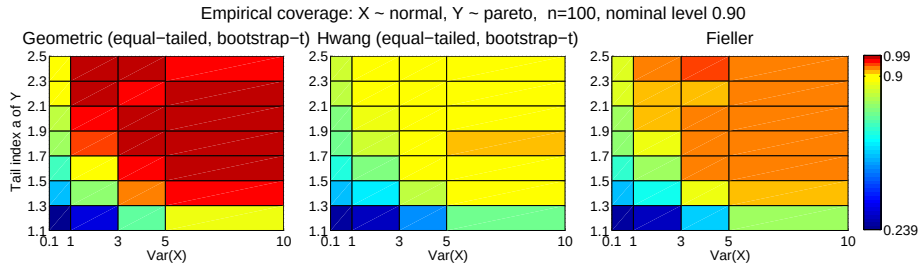


## Empirical coverage $X \sim \text{normal}, Y \sim \text{Pareto}, n=20$



## Empirical coverage

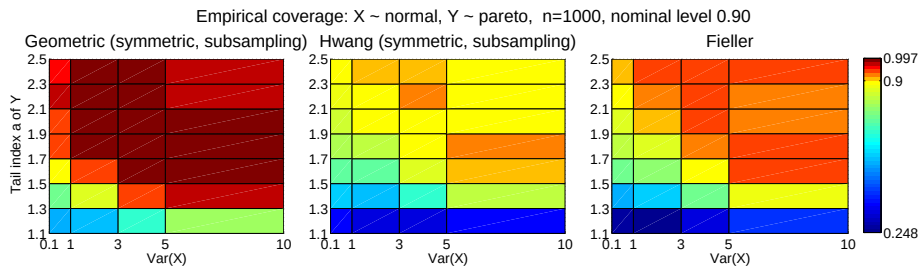
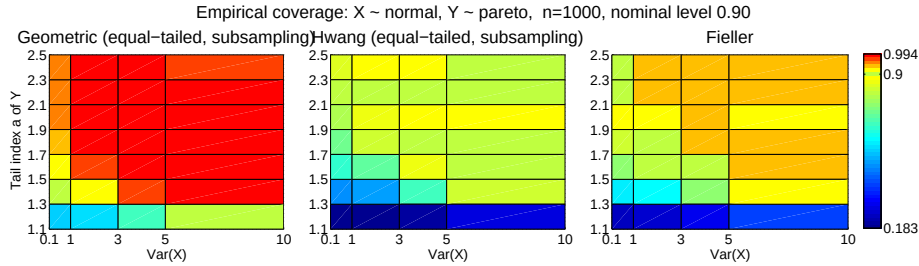
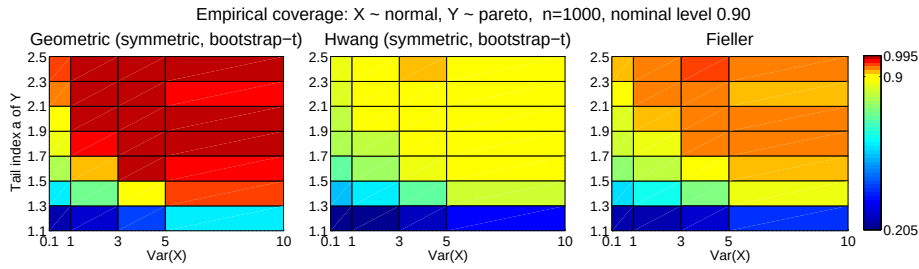
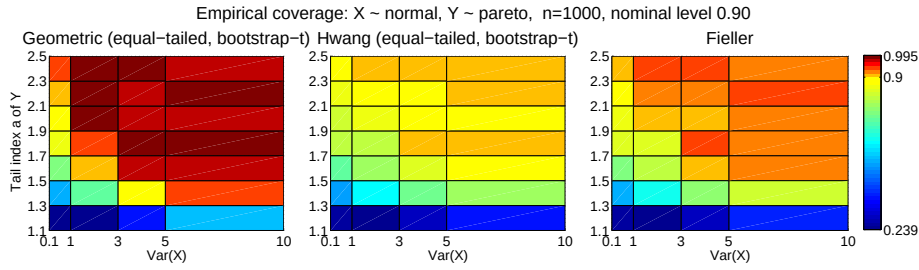
### $X \sim \text{normal}, Y \sim \text{Pareto}, n=100$





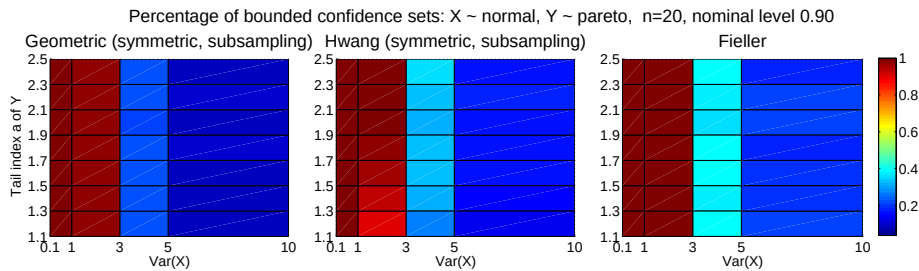
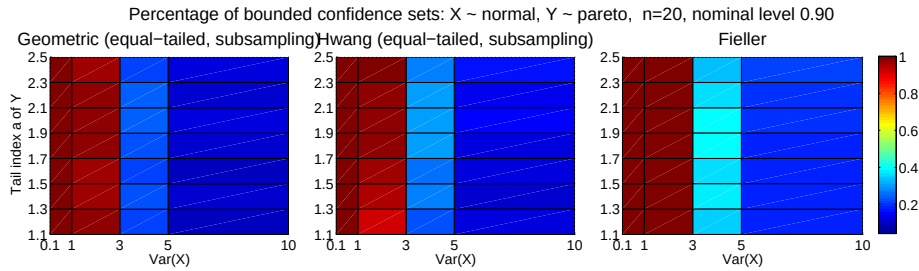
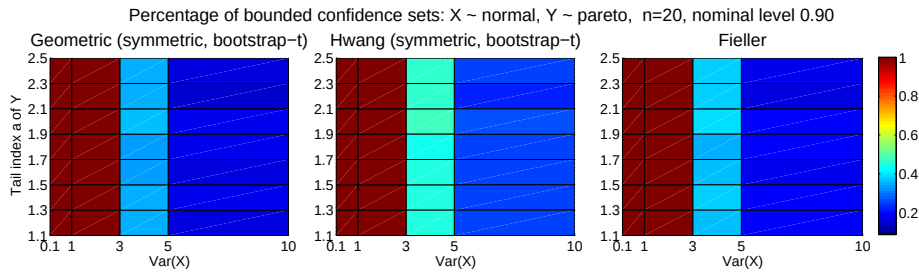
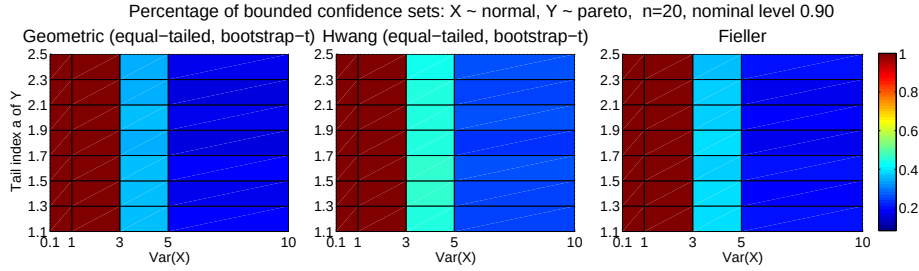
## Empirical coverage

$X \sim \text{normal}, Y \sim \text{Pareto}, n=1000$



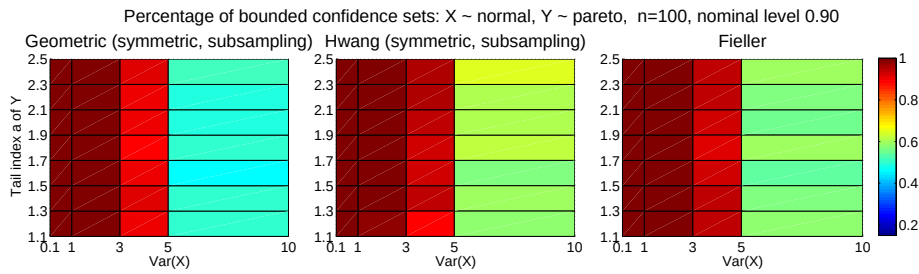
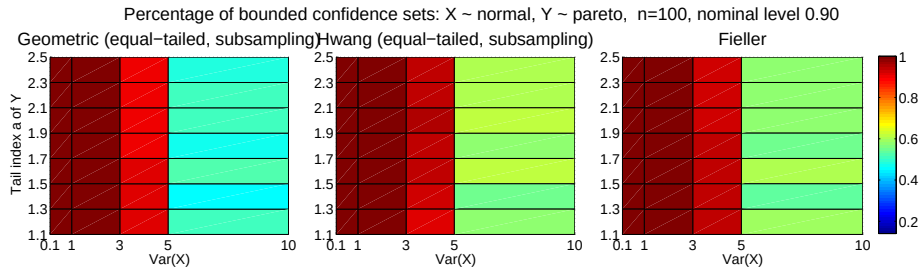
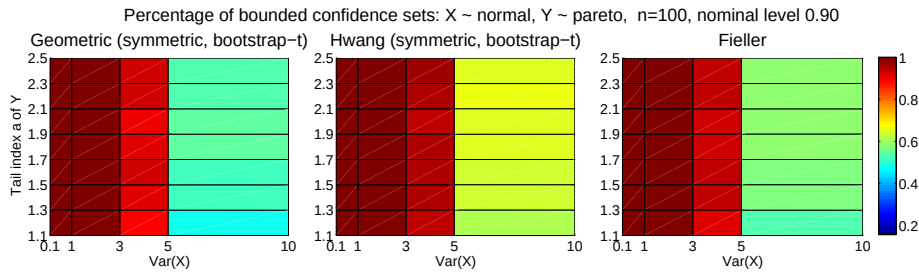
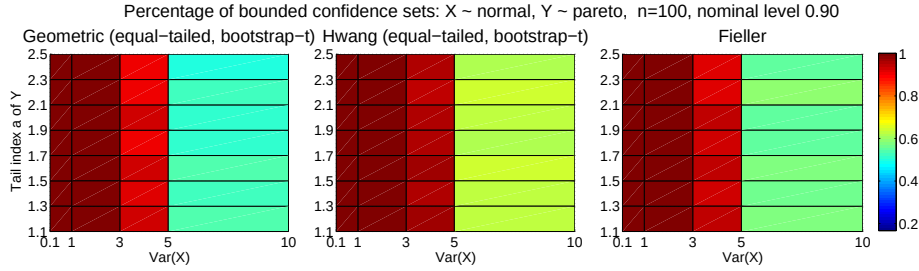
## Number of bounded confidence sets

$X \sim \text{normal}$ ,  $Y \sim \text{Pareto}$ ,  $n=20$



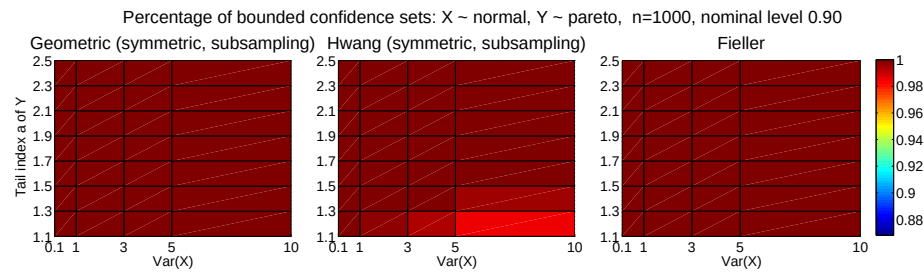
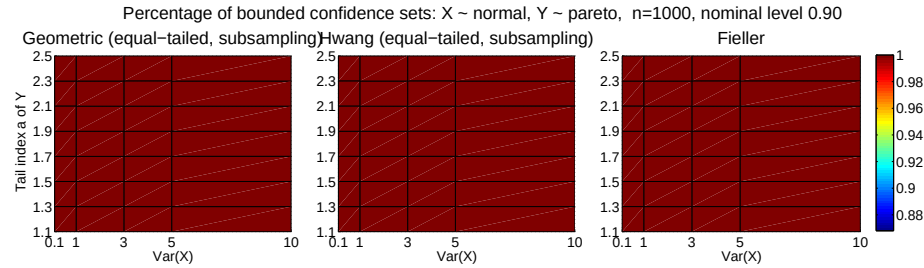
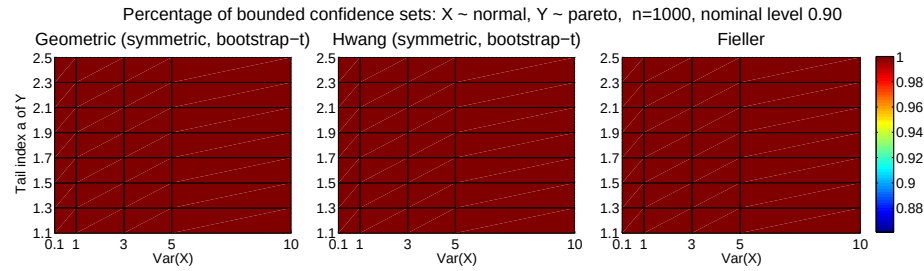
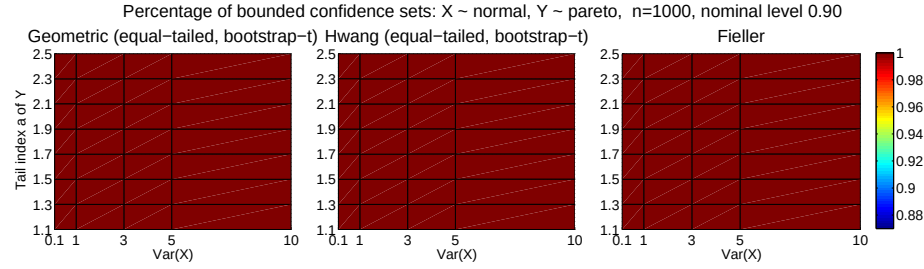
## Number of bounded confidence sets

$X \sim \text{normal}$ ,  $Y \sim \text{Pareto}$ ,  $n=100$



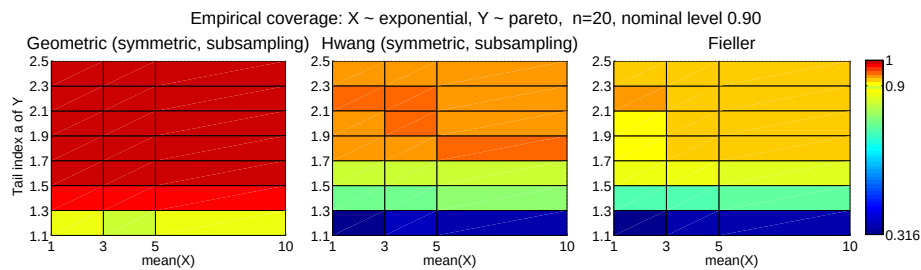
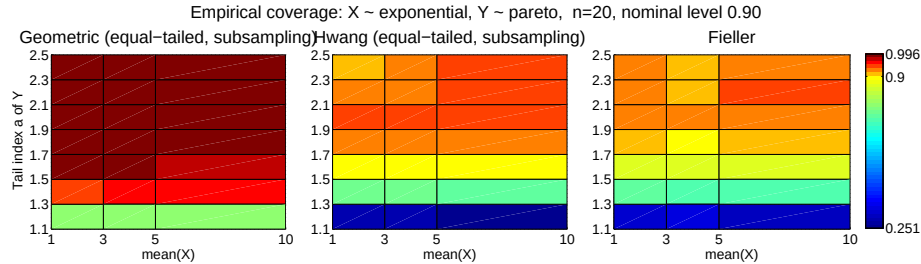
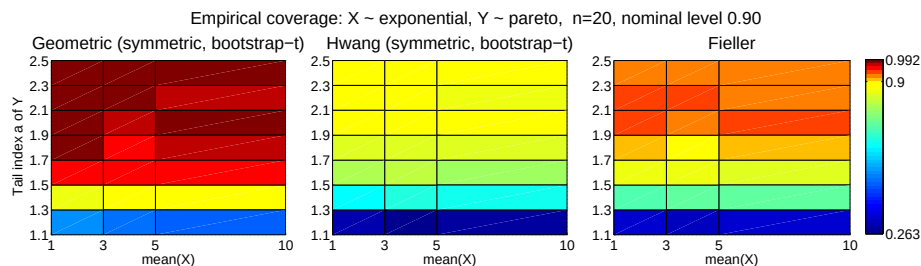
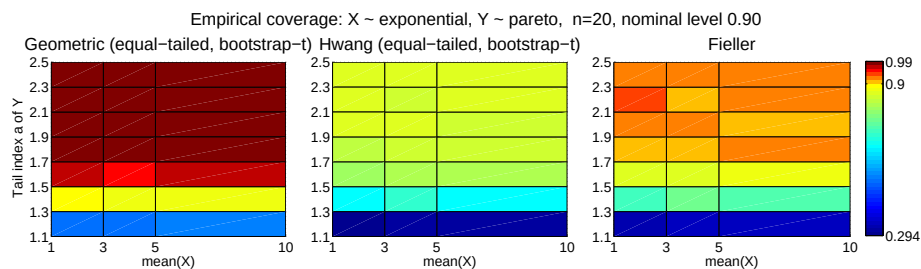
## Number of bounded confidence sets

$X \sim \text{normal}$ ,  $Y \sim \text{Pareto}$ ,  $n=1000$



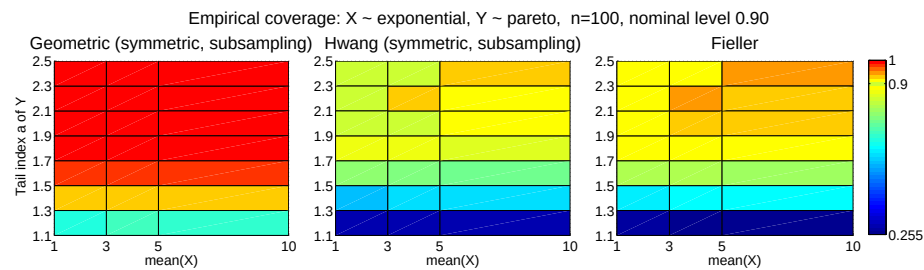
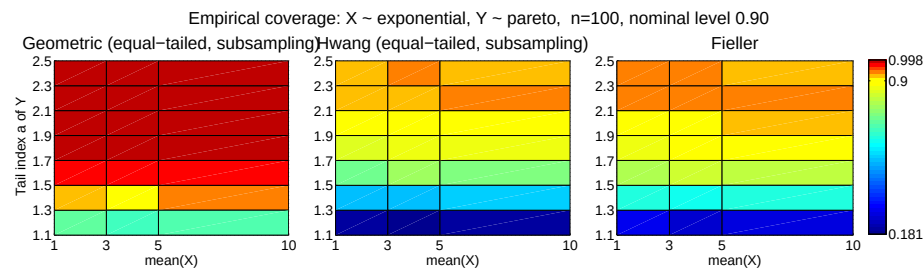
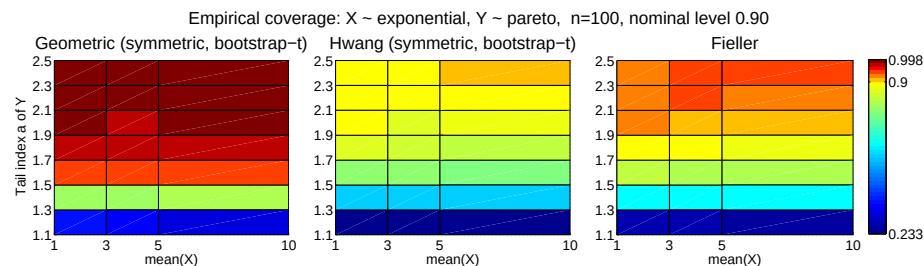
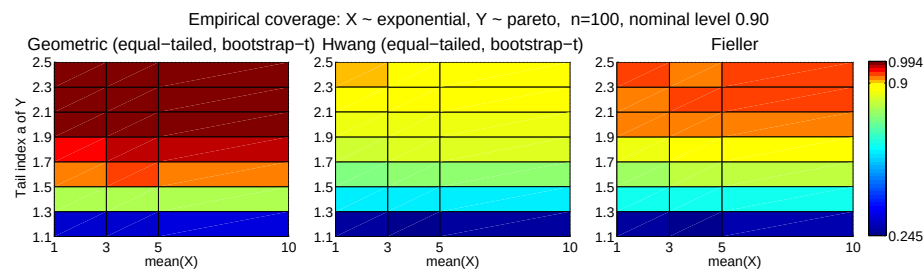
## Empirical coverage

$X \sim \text{exponential}$ ,  $Y \sim \text{Pareto}$ ,  $n=20$



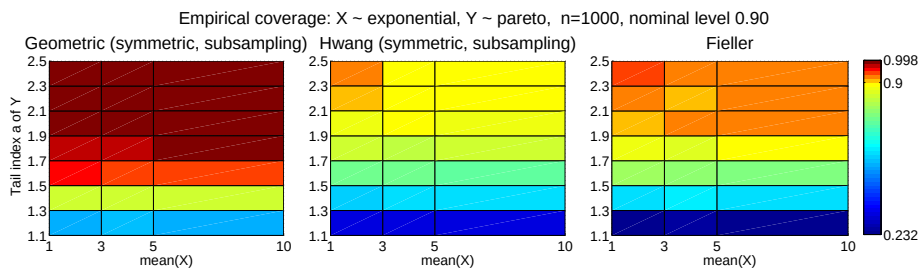
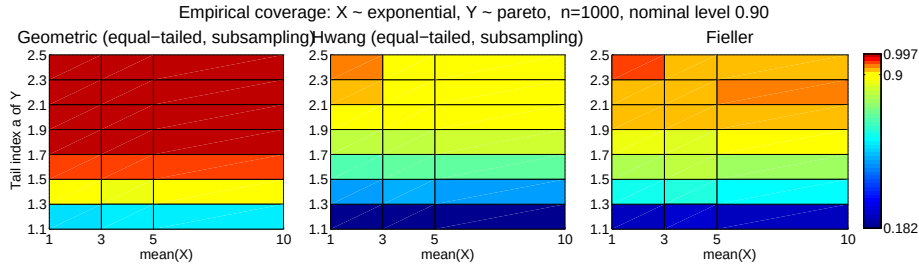
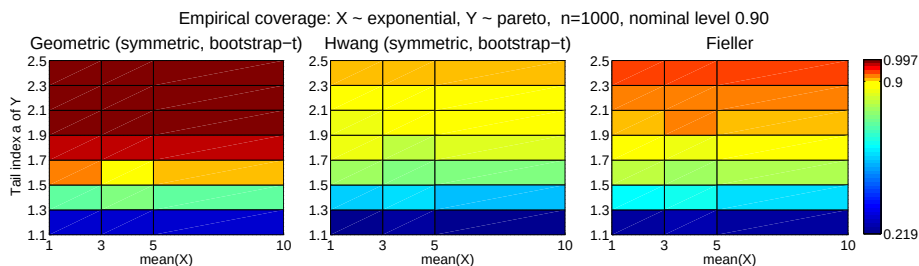
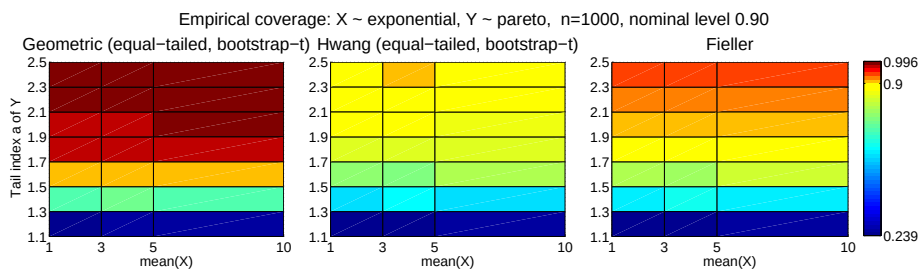
## Empirical coverage

### $X \sim \text{exponential}$ , $Y \sim \text{Pareto}$ , $n=100$

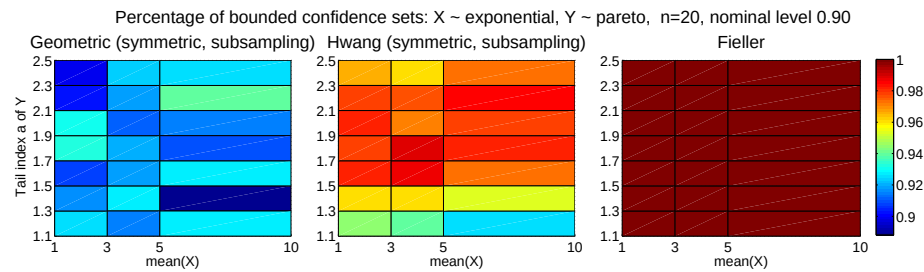
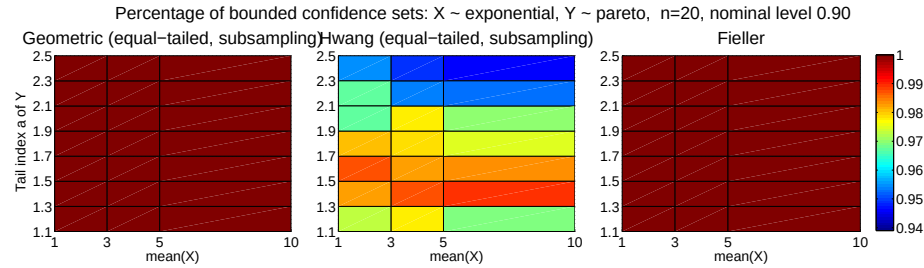
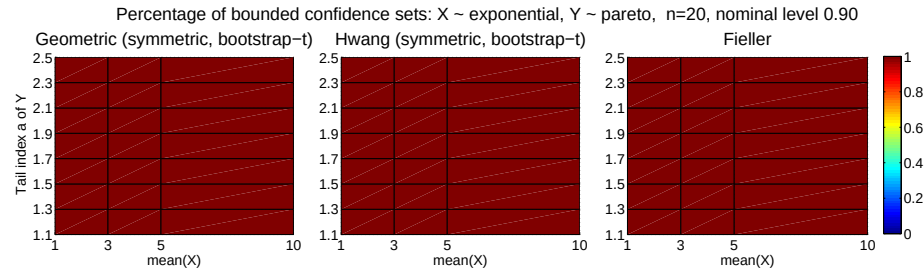
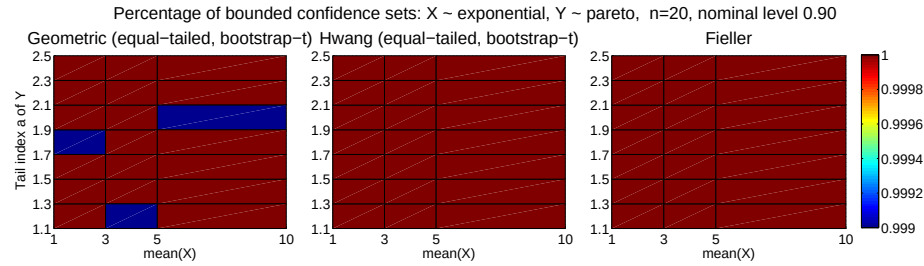


## Empirical coverage

### $X \sim \text{exponential}$ , $Y \sim \text{Pareto}$ , $n=1000$

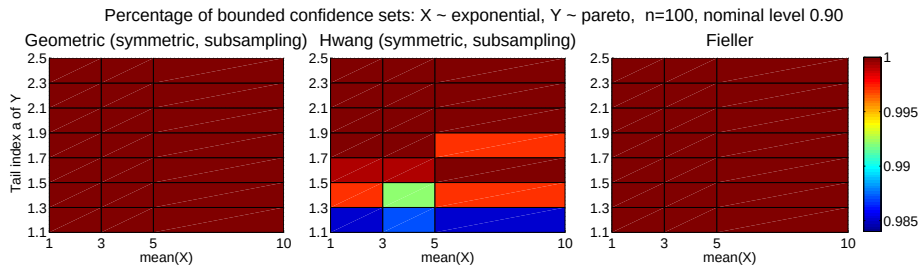
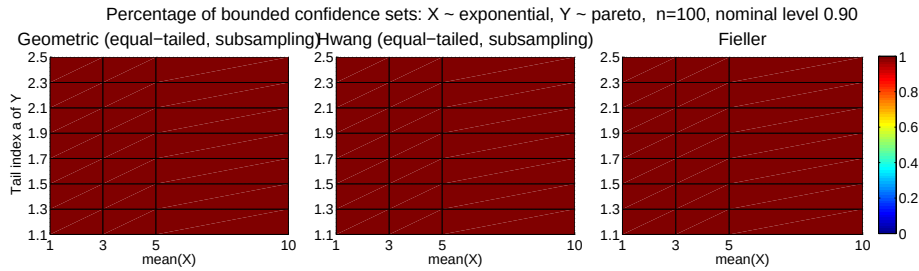
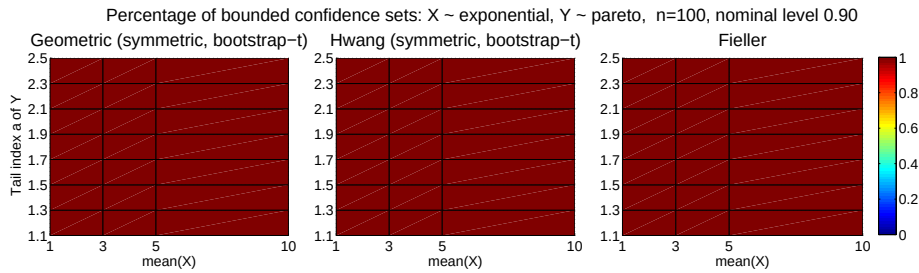
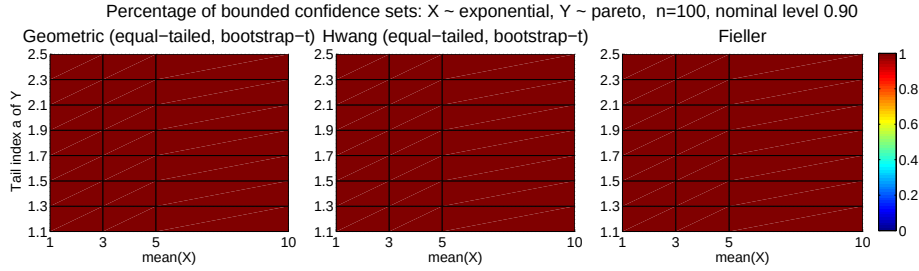


## Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{Pareto}$ , $n=20$

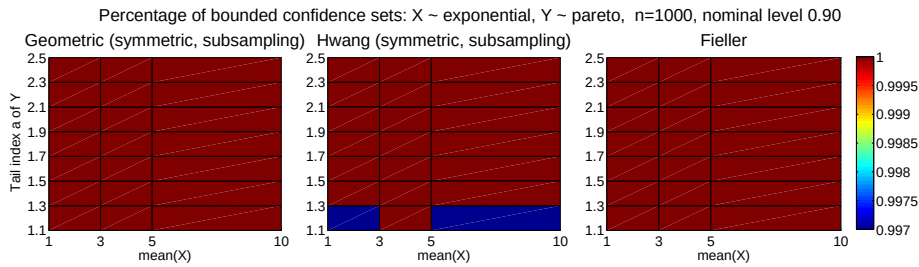
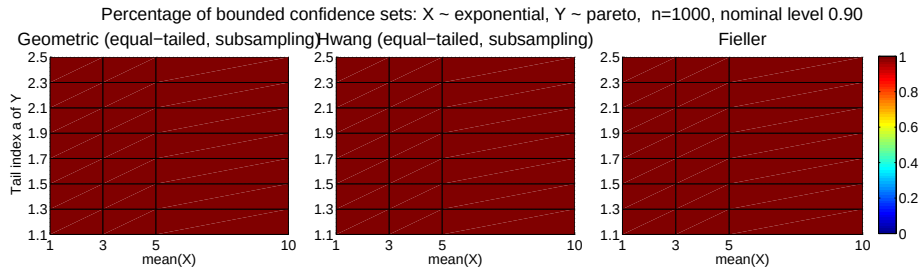
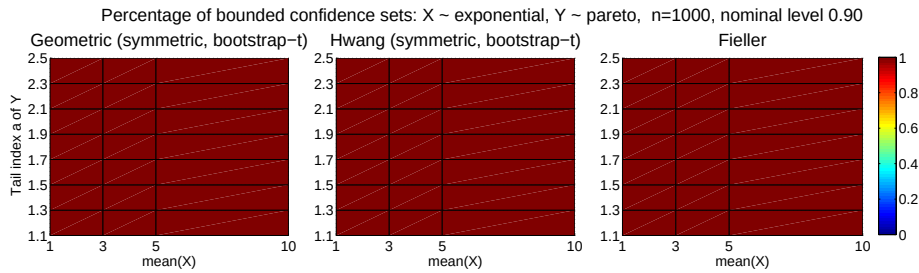
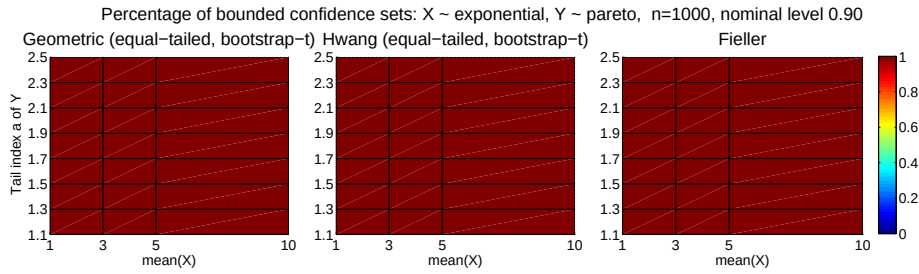




# Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{Pareto}$ , $n=100$

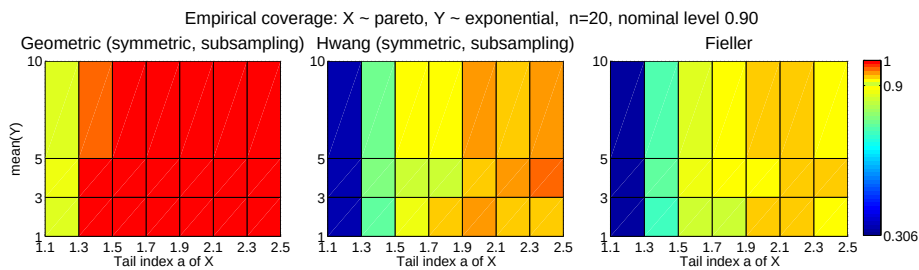
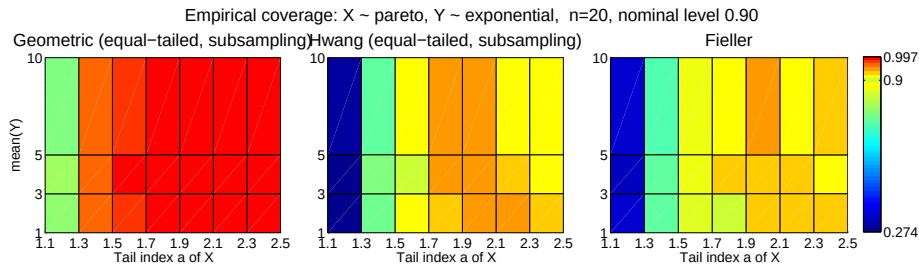
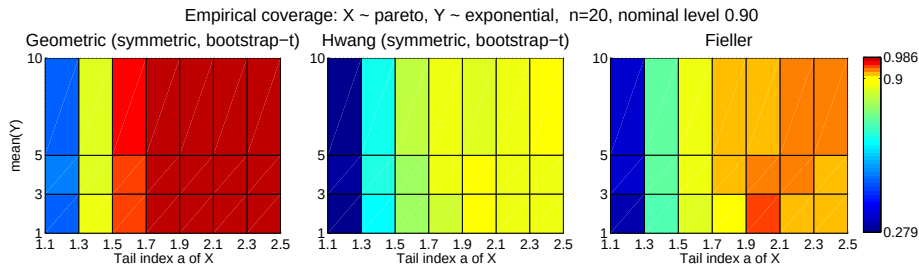
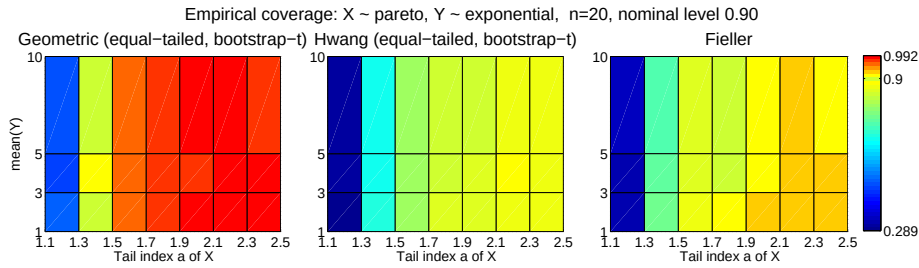


## Number of bounded confidence sets $X \sim \text{exponential}$ , $Y \sim \text{Pareto}$ , $n=1000$



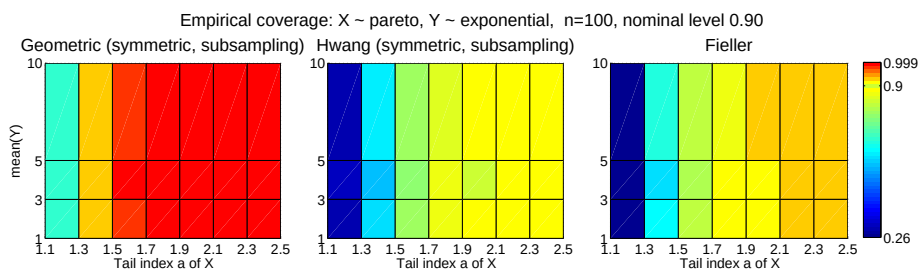
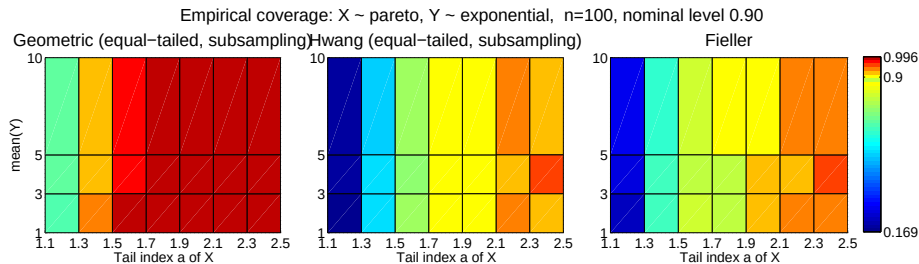
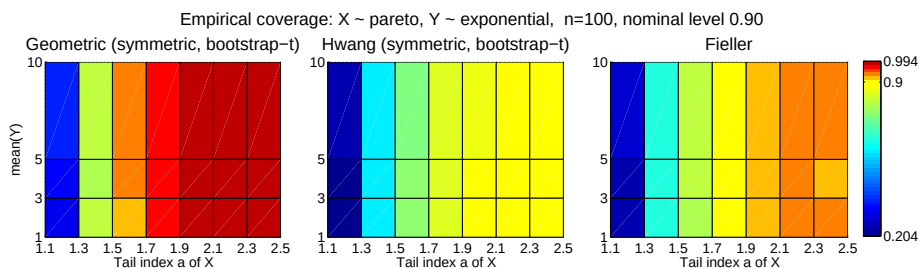
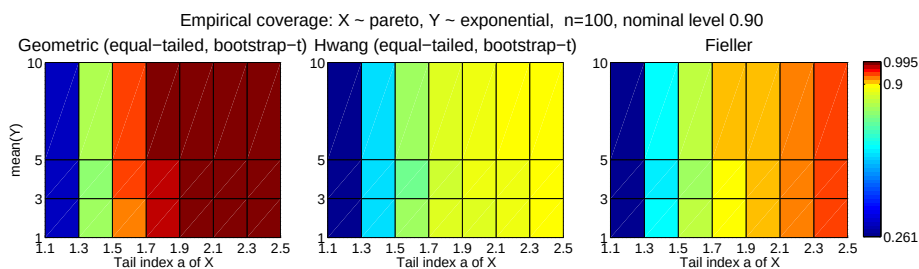
## Empirical coverage

### $X \sim \text{Pareto}$ , $Y \sim \text{exponential}$ , $n=20$

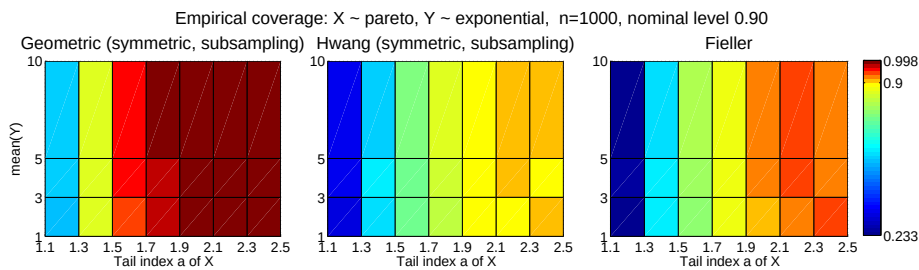
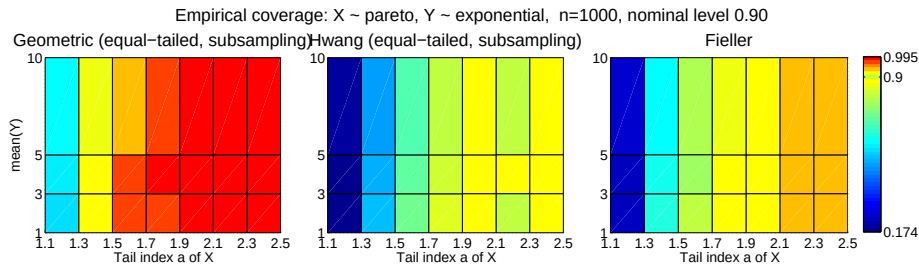
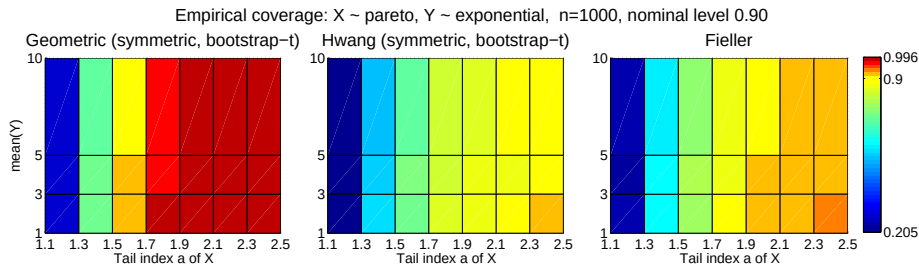
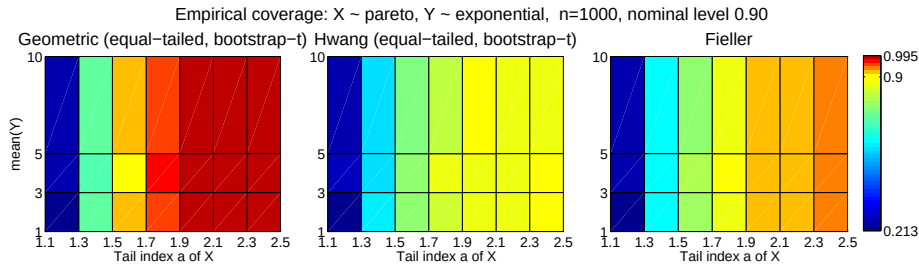


## Empirical coverage

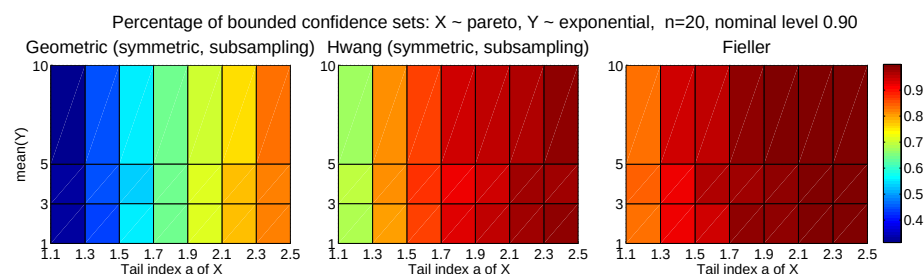
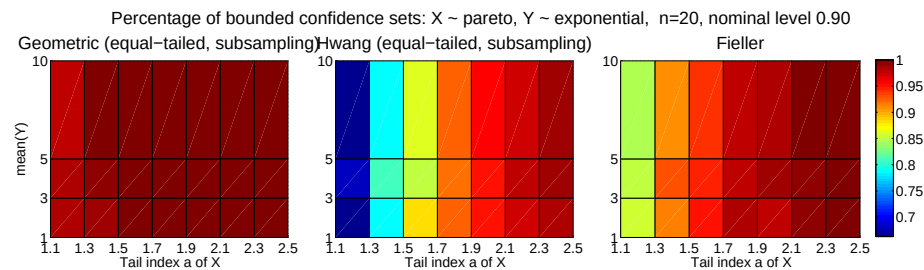
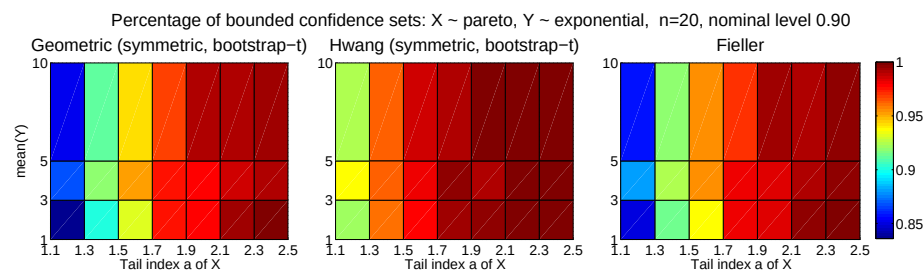
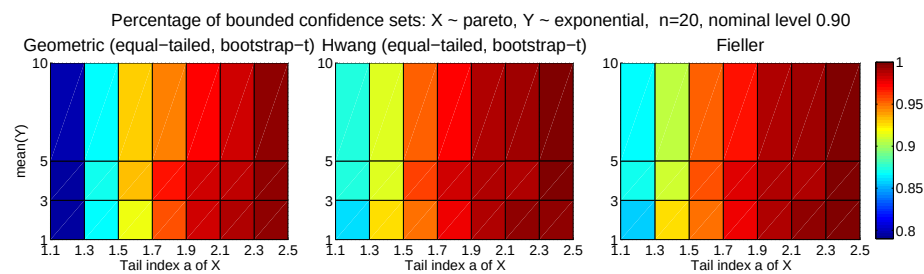
### $X \sim \text{Pareto}$ , $Y \sim \text{exponential}$ , $n=100$



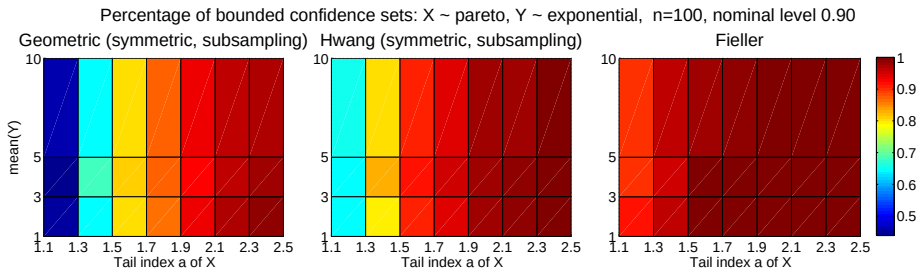
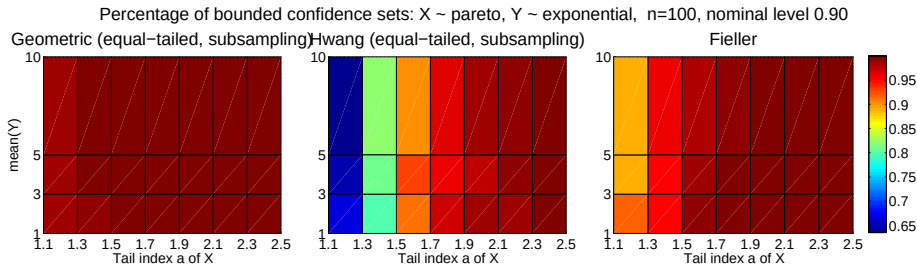
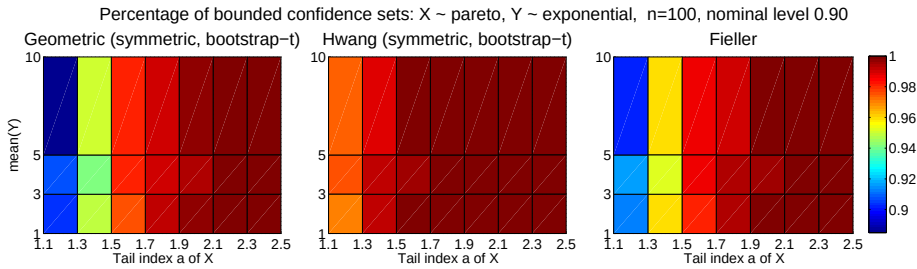
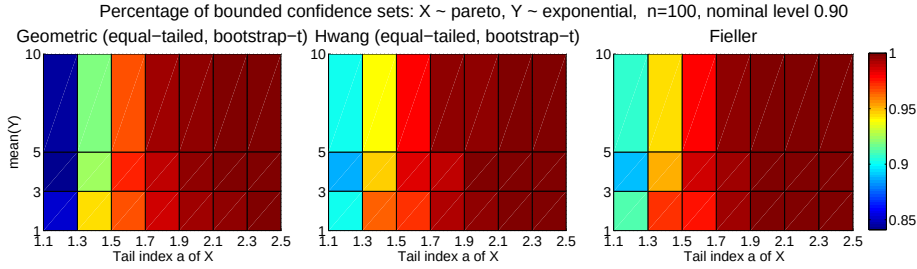
# Empirical coverage $X \sim \text{Pareto}$ , $Y \sim \text{exponential}$ , $n=1000$



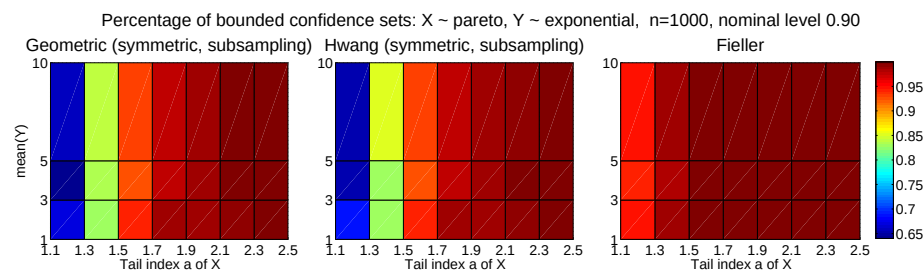
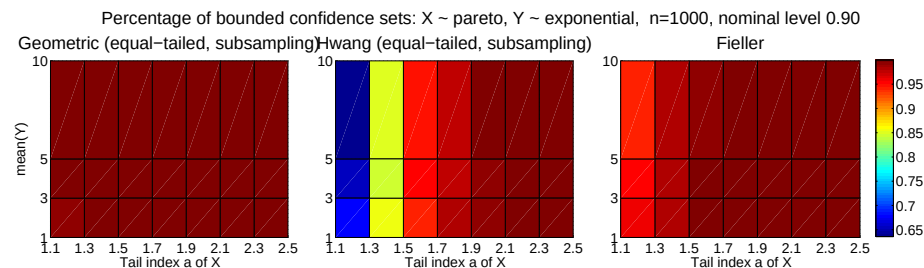
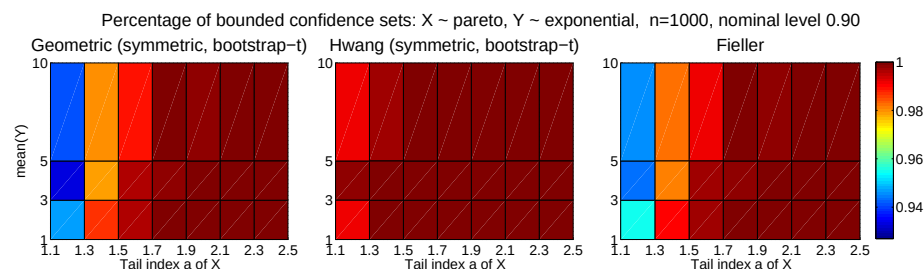
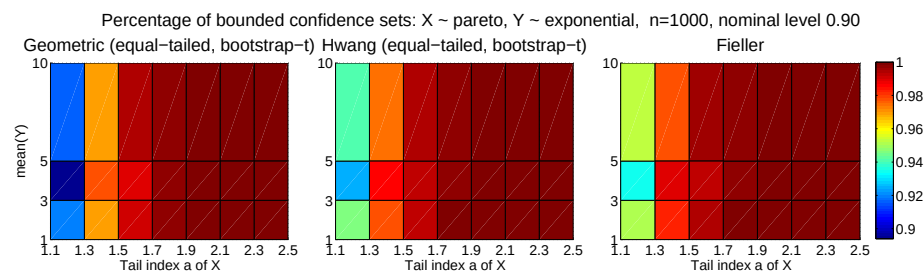
## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{exponential}$ , $n=20$



## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{exponential}$ , $n=100$

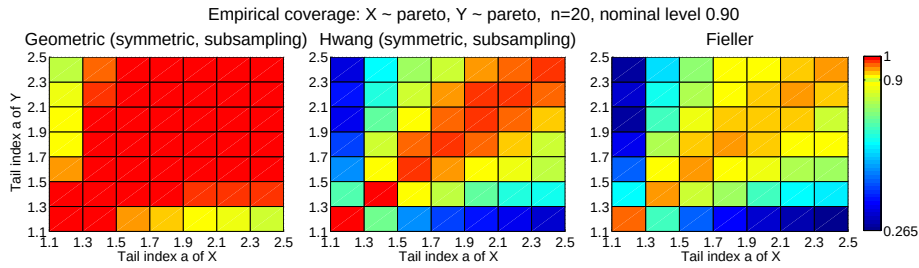
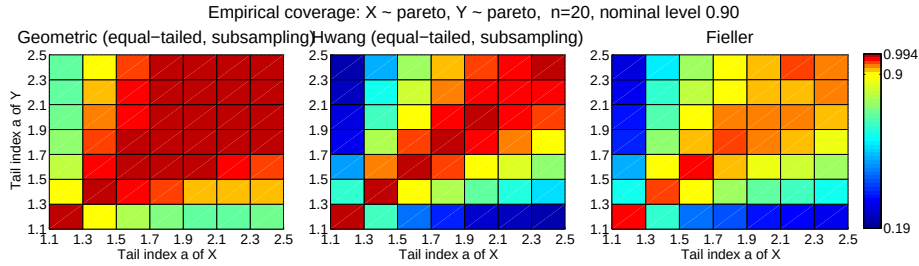
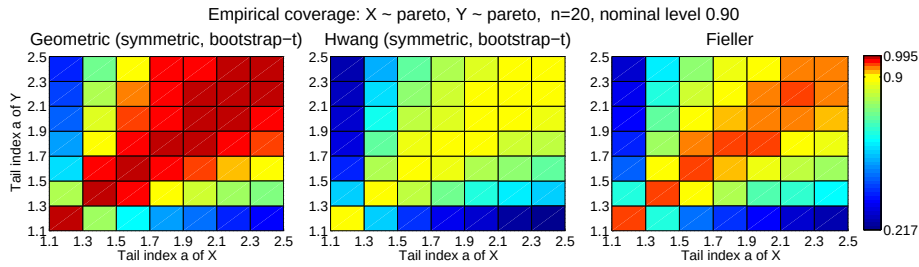
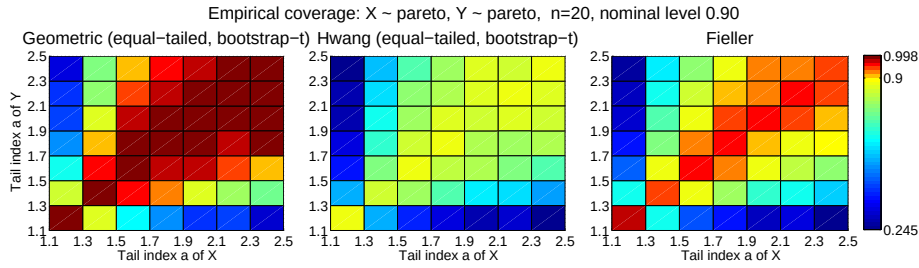


## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{exponential}$ , $n=1000$



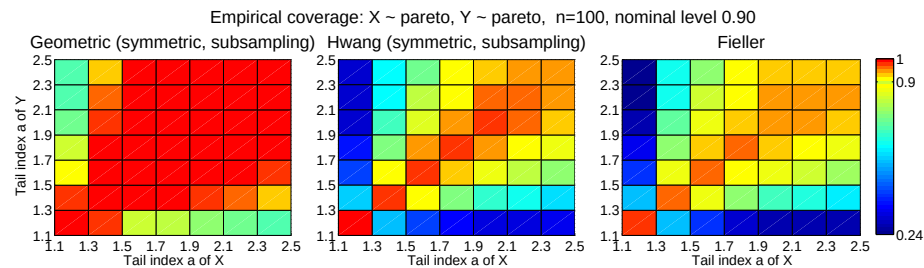
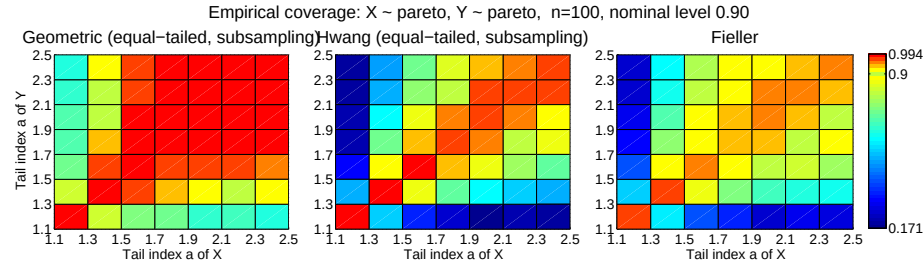
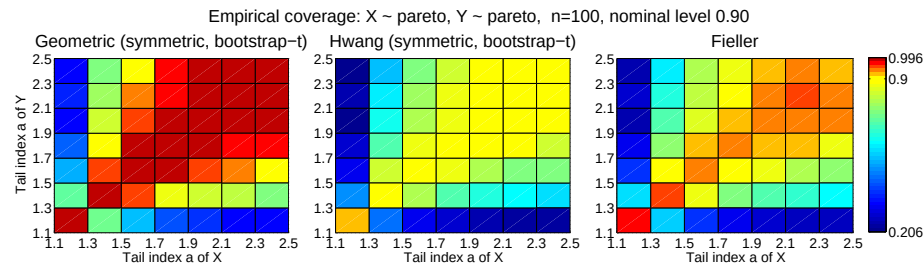
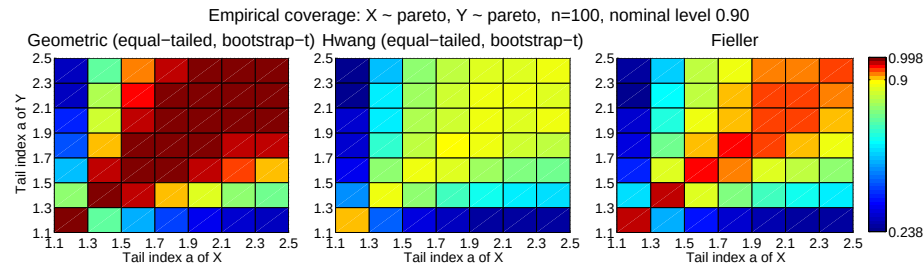


# Empirical coverage $X \sim \text{Pareto}, Y \sim \text{Pareto}, n=20$

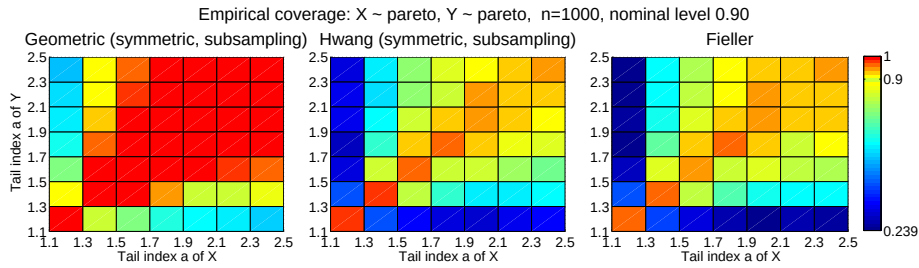
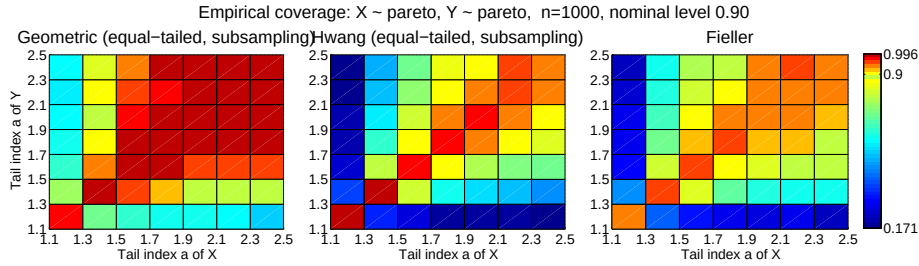
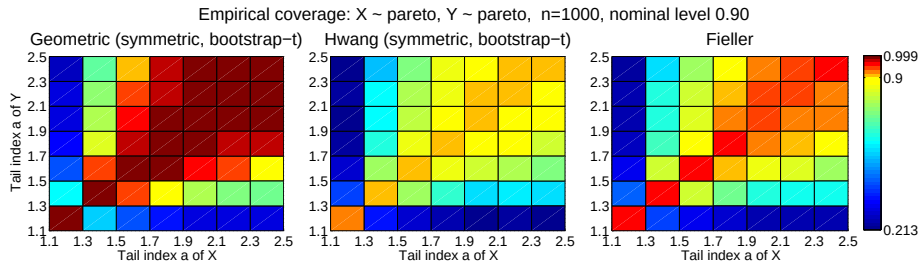
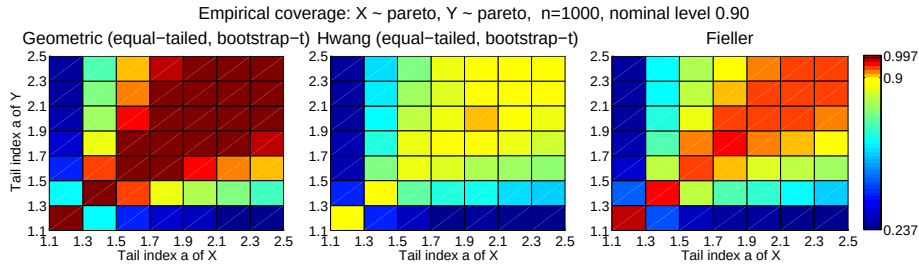


## Empirical coverage

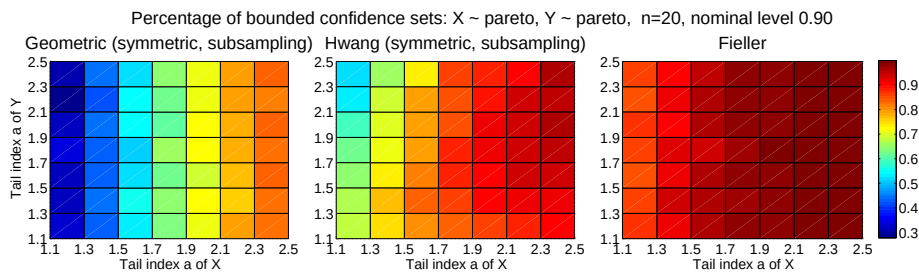
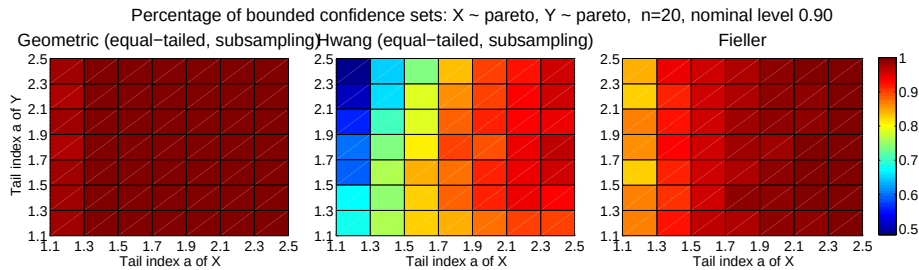
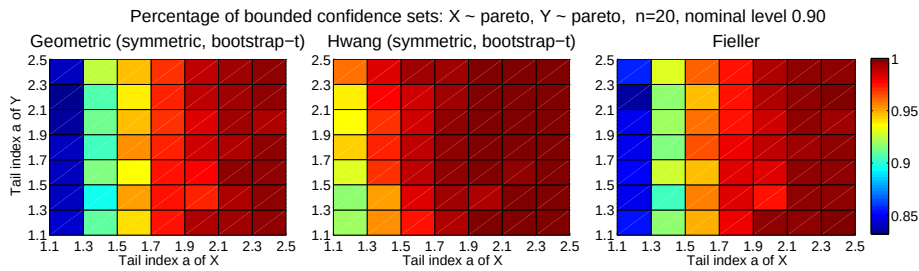
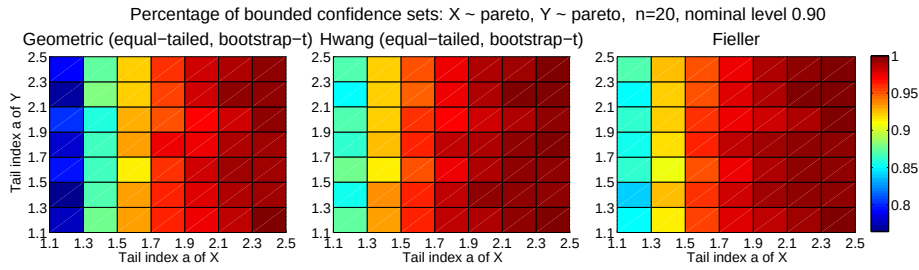
### $X \sim \text{Pareto}$ , $Y \sim \text{Pareto}$ , $n=100$



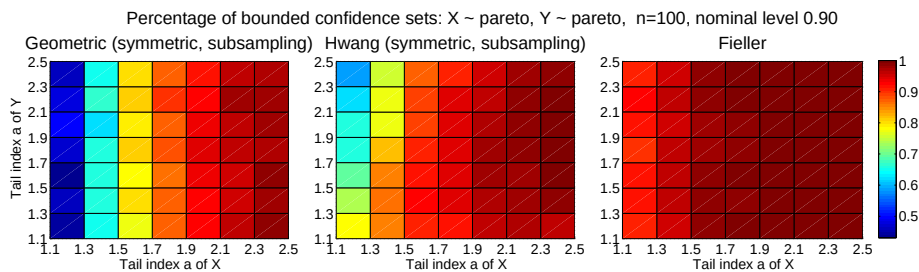
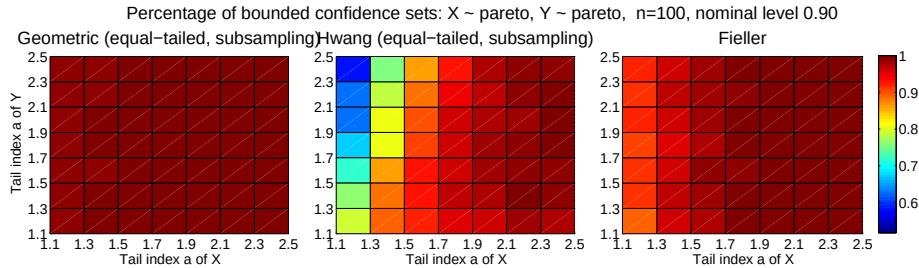
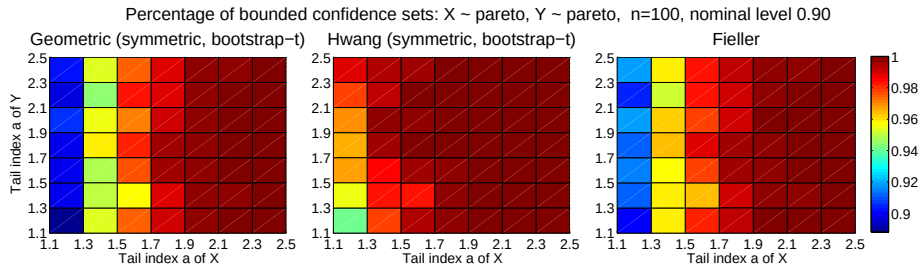
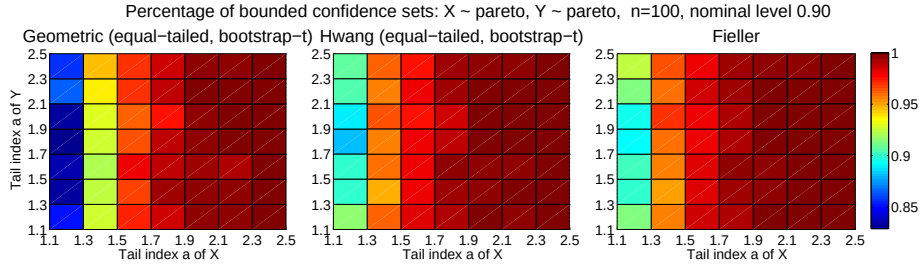
# Empirical coverage $X \sim \text{Pareto}, Y \sim \text{Pareto}, n=1000$



## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{Pareto}$ , $n=20$



# Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{Pareto}$ , $n=100$



## Number of bounded confidence sets $X \sim \text{Pareto}$ , $Y \sim \text{Pareto}$ , $n=1000$

